Chapter 4

Problem 55

\[
P(\text{no errors}) = P(\text{no errors|first typist}) P(\text{first typist}) + P(\text{no errors|second typist}) P(\text{second typist})
\]

\[
= \frac{1}{2} \left( \frac{3^0}{0!} e^{-3} + \frac{4.2^0}{0!} e^{-4.2} \right)
\]

\[
= \frac{1}{2} \left( e^{-3} + e^{-4.2} \right).
\]

Problem 57 \(X\) is Poisson with parameter \(\lambda = 3\).

(a) \(P\{X \geq 3\} = 1 - P\{X = 0\} - P\{X = 1\} - P\{X = 2\} = 1 - e^{-3} \left(1 + 3 + \frac{9}{2}\right) = 0.5768\).

(b) \(P\{X \geq 3|X \geq 1\} = \frac{P\{X \geq 3\}}{P\{X \geq 1\}} = \frac{P\{X \geq 3\}}{1 - e^{-3}} = 0.6070\).

Problem 59 Let \(X\) be the number of times you win a prize. Then \(X\) is binomial with \(n = 50\) and \(p = \frac{1}{100}\), i.e., we can use the Poisson approximation with \(\lambda = 50 \cdot \frac{1}{100} = \frac{1}{2}\).

(a) \(P\{X \geq 1\} = 1 - P\{X = 0\} = 1 - e^{-\frac{1}{2}} = 0.3935\)

(b) \(P\{X = 1\} = \frac{1}{2} e^{-\frac{1}{2}} = 0.3033\)

(c) \(P\{X \geq 2\} = 1 - P\{X = 0\} - P\{X = 1\} = 1 - e^{-\frac{1}{2}} \left(1 + \frac{1}{2}\right) = 0.0902\)

Problem 61 Let \(X\) be Poisson with parameter \(\lambda = 1000 \cdot 0.0014 = 1.4\). Then \(P\{X \geq 2\} = 1 - P\{X = 0\} - P\{X = 1\} = 1 - e^{-1.4}(1+1.4) = 0.4082\).

Problem 63 Let \(X\) be a Poisson random variable with parameter \(\lambda = \frac{5}{2}\). Then \(X\) gives a reasonable description of the number of people entering the casino between 12 and 12:05.

(a) \(P\{X = 0\} = e^{-\frac{5}{2}} = 0.0821\)
(b) \( P \{ X \geq 4 \} = 1 - e^{-\frac{5}{2}} \left( 1 + \frac{5}{2} + \frac{25}{8} + \frac{125}{48} \right) = 0.2424 \)

**Problem 72** Let \( A \) be the stronger team. \( P (A \text{ wins in } i \text{ games}) = \binom{i-1}{i-4} 0.6^i 0.4^{i-4}, \) for \( i = 4, \ldots, 7. \) Hence

\[
P (A \text{ wins best-of-seven series}) = \sum_{i=4}^{7} \binom{i-1}{i-4} 0.6^i 0.4^{i-4} = 0.7102.
\]

Similarly,

\[
P (A \text{ wins best-of-three series}) = \sum_{i=2}^{3} \binom{i-1}{i-2} 0.6^i 0.4^{i-2} = 0.6480.
\]

**Problem 73** Let \( X \) be the number of games played in a match. Then \( P \{ X = i \} = 2 \binom{i-1}{i-4} \left( \frac{1}{2} \right)^i \) for \( i = 4, \ldots, 7. \) Hence, \( E[X] = 2 \sum_{i=4}^{7} i \binom{i-1}{i-4} \left( \frac{1}{2} \right)^i = 5.8125. \)

**Problem 77** Let \( E \) be the event that right-hand box is emptied while the left-hand box still contains \( k \) matches. Then, using a negative binomial random variable with \( p = \frac{1}{2}, \ r = N, \) and \( n = 2N - k, \) we see that \( P (E) = \binom{2N-k-1}{N-1} \left( \frac{1}{2} \right)^{2N-k}. \) Now the desired probability is \( 2 P (E). \)

**Problem 78** Let \( E \) be the event that a single drawing results in two white and two black balls. Then \( P (E) = \frac{\binom{1}{4} \binom{1}{4}}{\binom{4}{4}} = \frac{18}{35}. \)

Let \( X \) be the number of selections until \( E \) occurs. Then

\[
P \{ X = n \} = \frac{17^{n-1} \cdot 18}{35^n}.
\]

**Problem 79**

(a) \( P \{ X = 0 \} = \frac{\binom{94}{10}}{\binom{100}{10}} = 0.5223 \)

(b)

\[
P \{ X > 2 \} = 1 - P \{ X = 0 \} - P \{ X = 1 \} - P \{ X = 2 \}
= \frac{\binom{100}{10} - \binom{94}{10} - \binom{6}{1} \binom{94}{9} - \binom{6}{2} \binom{94}{8}}{\binom{100}{10}} = 0.0126
\]
Problem 84 (a) For \(i = 1, \ldots, 5\), let \(X_i = 1\) if the \(i\)-th box is empty and \(X_i = 0\) otherwise. Then \(X = X_1 + \cdots + X_5\) is the number of empty boxes. For \(i = 1, \ldots, 5\),

\[
E[X_i] = P(X_i = 1) = (1 - p_i)^{10}.
\]

Thus

\[
E[X] = E[X_1] + \cdots + E[X_5] = \sum_{i=1}^{5} (1 - p_i)^{10}.
\]

(b) For \(i = 1, \ldots, 5\), let \(Y_i = 1\) if the \(i\)-th box has exactly 1 ball and \(Y_i = 0\) otherwise. Then \(Y = Y_1 + \cdots + Y_5\) is the number of boxes that have exactly 1 ball. For \(i = 1, \ldots, 5\),

\[
E[Y_i] = P(Y_i = 1) = 10p_i (1 - p_i)^9.
\]

Thus

\[
E[Y] = E[Y_1] + \cdots + E[Y_5] = \sum_{i=1}^{5} 10p_i (1 - p_i)^9.
\]

Problem 85 For \(i = 1, \ldots, k\), let \(X_i = 1\) if the \(i\)-th type appear at least once in the set of \(n\) coupons. Then \(X = X_1 + \cdots + X_k\) is the number of distinct types that appear in this set. For \(i = 1, \ldots, k\),

\[
E[X_i] = P(X_i = 1) = 1 - P(X_i = 0) = 1 - (1 - p_i)^n.
\]

Thus

\[
E[X] = E[X_1] + \cdots + E[X_k] = k - \sum_{i=1}^{k} (1 - p_i)^n.
\]