4.  
   a) Five balls  
   b) Thirteen balls  

16.  5  
( for the complete solution, follow steps from the lecture notes  
   to § 6.2, page  

22. Applying Thm 3,  
    from the book, pg. 403  

36. Suppose in the group of 6 computers, there are no 2 computers  
    which are directly connected to the same no. of computers.  
    There are 6 computers, each computer is directly connected to  
    at least one of the other computers, we have that each  
    computer can be connected to either 1, 2, 3, 4 or 5  
    computers. Hence there are 5 options. But there are 5 computers in total, so, at  
    least 2 computers will have the same option, i.e., they  
    will be connected to the same no. of computers.  

46. Suppose, for all \( i = 1, 2, \ldots, t \), the \( i \)th box  
    contains less than \( n_i \) object, i.e., 1st box contains  
    less than \( n_1 \) objects, 2nd box contains less than \( n_2 \)  
    objects, & so on.  
    Then all the \( t \) boxes together contain not more than  
    \( (n_1-1) + (n_2-1) + \cdots + (n_t-1) \) object  

Now  
\[
(n_1-1) + (n_2-1) + \cdots + (n_t-1) = n_1 + \cdots + n_t - t
\]

But the total no. of objects  

\( n_1 + n_2 + \cdots + n_t + t + 1 \)  

to be placed into \( t \) boxes are  

Thus 1 object is left, hence a contradiction  

& so for some \( i, i = 1, 2, \ldots, t \), the \( i \)th box contains at least  

\( n_i \) objects.  

§ 6.3

8. \( P(5, 5) = 5 \times 4 \times 3 \times 2 \times 1 = 5! \)

12. a) Bit strings are made of either 0s or 1s.

\( \text{No. of ways of choosing 3 places out of 12 for putting 1s} = \binom{12}{3} = \frac{12!}{3! \cdot 9!} \)

b) 0 Zero 1 s = 1 way

One 1 s = \( \binom{12}{1} = \frac{12!}{1! \cdot 11!} = 12 \)

Two 1 s = \( \binom{12}{2} = \frac{12!}{2! \cdot 10!} \)

Three 1 s = \( \binom{12}{3} = \frac{12!}{3! \cdot 9!} \)

No. of ways of bit strings of length 12 containing at least three 1 s

\[ = 1 + \binom{12}{1} + \binom{12}{2} + \binom{12}{3} + \binom{12}{4} + \binom{12}{5} + \binom{12}{6} + \binom{12}{7} + \binom{12}{8} + \binom{12}{9} + \binom{12}{10} + \binom{12}{11} + \binom{12}{12} \]

c) No. of bit strings of length 12 containing at least three 1 s

\[ = \binom{12}{3} + \binom{12}{4} + \binom{12}{5} + \binom{12}{6} + \binom{12}{7} + \binom{12}{8} + \binom{12}{9} + \binom{12}{10} + \binom{12}{11} + \binom{12}{12} \]

d) Equal number of 0s and 1s = \( \binom{12}{6} + \binom{12}{6} \)

16. For a set with 10 elements can have either \( \binom{10}{1} \) element subset.

16. A subset of a set with 10 elements should have either 1 element or 3 or 5 or 7 or 9 elements so it to have an odd number of elements.

No. of subsets of a set with 10 elements with 1 element

\[ = \binom{10}{1} \]

\[ = \binom{10}{3} \]
With 5 elements = \( C(10, 5) \)
with 7 " = \( C(10, 7) \)
" 9 elements = \( C(10, 9) \)

Thus, total no. of subsets with an odd number of elements
= \( C(10, 1) + C(10, 3) + C(10, 5) + C(10, 7) + C(10, 9) \)

18. a) \( 2^8 \)
b) \( C(8, 3) \)
c) \( C(8, 3) + C(8, 4) + C(8, 5) + C(8, 6) + C(8, 7) + C(8, 8) \)
d) \( C(8, 4) \)

22. a) Since ED occurs as a block, we permute only 7 letters,
ED, A, B, C, F, G, H = \( P(7, 7) = 7! \)
b) CDE, A, B, F, G, H = \( P(6, 6) = 6! \)
c) BA, FGH, C, D, E = \( P(5, 5) = 5! \)
d) AB, DE, G, H, C, F = \( P(5, 5) = 5! \)
e) CAB, BCABED, CABED, F, G, H = \( P(4, 4) \cdot 4! \)
f) 0 (since we cannot have both the strings BCA and ABF together)

40. Refer to example in lecture notes for Section 6.3, page 4.
§ 6.4.
8. \((3x + 2y)^{17} = \sum_{k=0}^{17} \binom{17}{k} 3^k x^{17-k} 2^k y^k\)

Coefficient of \(x^8 y^9 = \binom{17}{8} 3^8 2^9\)

= \(\binom{17}{8} 3^8 2^9\)

12. \(\binom{10}{0} \binom{10}{1} \binom{10}{2} \binom{10}{3} \binom{10}{4} \binom{10}{5} \binom{10}{6} \binom{10}{7} \binom{10}{8} \binom{10}{9} \binom{10}{10}\)

In the form of binomial coefficients, it is.

\[\binom{10}{0} + \binom{10}{1} = \binom{11}{1}\]

\[(\sin (\frac{n}{k}) + (\frac{n}{k}) = (\frac{n+1}{k})\]

Hence the next row following Row 0 is:

\[\binom{11}{0} \binom{11}{1} \binom{11}{2} \binom{11}{3} \binom{11}{4} \binom{11}{5} \binom{11}{6} \binom{11}{7} \binom{11}{8} \binom{11}{9} \binom{11}{10}\]

14. Clearly \(\binom{n}{0} = 1\)

We first show: \(\text{If } \frac{k}{n} \in \mathbb{N} \), then

\(\binom{n}{k} = \binom{\frac{k}{n+1}}{k+1}\)

We first show: \(\text{If } 1 \leq k \leq \left\lfloor \frac{n}{2} \right\rfloor \), then

\(\binom{n}{k-1} < \binom{n}{k} \quad \text{(D)}\)

Now,

\(\binom{n}{k-1} = \frac{n!}{(k-1)! (n-(k-1))!} = \frac{n!}{k (k-1)! (n-k+1) (n-k)!} = \binom{k}{n} (\frac{n}{k+1})\)

Since \(k < \left\lfloor \frac{n}{2} \right\rfloor \Rightarrow k < \frac{n}{2} \Rightarrow k < \frac{n+1}{2} \)

and so \(\left(\frac{k}{n+1}\right) < 1\)
This shows \( \binom{n}{k-1} = \frac{k}{n-k+1} \binom{n}{k} < \binom{n}{k} \)

Similarly, if \( \frac{1}{2} k < n-1 \), then

\[
\binom{n}{k} > \binom{n}{k+1}
\]

Since, in this case, \( 1 < n-k < \left\lfloor \frac{n}{2} \right\rfloor \) & so from 10:

\[
\binom{n}{n-k-1} < \binom{n}{n-k} - 10
\]

but \( \binom{n}{n-k} = \binom{n}{k} \) and

\[
\binom{n}{n-k-1} = \binom{n}{k+1}
\]

Therefore 10 gives

\[
\binom{n}{k+1} < \binom{n}{k}.
\]

28. b) To show: \( \binom{2n}{2} = 2\binom{n}{2} + n^2 \)

Sol.: Right side = \( 2 \frac{n!}{(n-2)!2!} + n^2 \)

Using Vandermonde's identity for \( m = n \) & \( k = 2 \).

\[
\binom{2n}{2} = \sum_{k=0}^{2} \binom{n}{2-k} \binom{n}{k}
\]

\[= \binom{n}{2}(0) + \binom{n}{1}(1) + \binom{n}{0}(2) \]

Now \( \binom{n}{0} = 1 \) & \( \binom{n}{1} = n \)

\[= 2\binom{n}{2} + n^2 \]