Final Schedule:
**Final: 5/9, Friday, 1:30PM-4:30PM, 114 David Kinley.**
Final make-up: 5/9, Friday, **8AM-11AM**, 141 Altgeld Hall.

**You must carry the University ID with you.**

The final exam is cumulative. There will be 10 multiple-choice problems (10 points each), and 5 logical-reasoning problems (20 points each). The following gives an incomplete list of the materials we have learned this semester. I strongly suggest you understand the problems in the first three midterms.

Chapter 1: Truth value of propositions

Chapter 2: Set cardinality, set operations, set identities, function properties, function operations

Chapter 3: Common algorithms, algorithm complexity

Chapter 5: Mathematical induction, strong induction

Chapter 6: Counting rules, pigeonhole principle, permutation, combination, binomial coefficients and identities

Chapter 7: Conditional probability, independent event, Bayes’ theorem

Chapter 8: Finding and solving recurrence relation, inclusion-exclusion principle

Chapter 9: Relations and representations, equivalence relations

Chapter 10: Graphs and representations, bipartite graphs, graph connectivity
The Binary Search algorithm is as follows:

**Procedure** Binary Search($x$ : positive integer, $a_1, \ldots , a_n$ : distinct integers)

\[
i := 1 \\
j := n
\]

\textbf{while} $i < j$

\[
m := \lfloor (i + j)/2 \rfloor
\]

if $x > a_m$ then $i := m + 1$
else $j := m$

if $x = a_i$ then location := $i$
else location := 0

\textbf{return} location.

What is the complexity measured in terms of operations?
Problem. Show that $3x^2 + 8x \log(x^{10})$ is $\Theta(x^2)$. 
Problem. How many injective functions are there from a set with 4 elements to a set with 100 elements?
Problem. Let $S$ be the set of all functions from $\mathbb{Z}$ to $\mathbb{Z}$, determine whether the following relation is an equivalence relation:
(a) $\{(f, g) \mid f(0) = g(0) \text{ and } f(1) = g(1)\}$.
(b) $\{(f, g) \mid f(0) = g(1) \text{ and } f(1) = g(0)\}$. 
Problem. Determine the truth value of the following proposition:

$$\forall x \left( (x = 1) \leftrightarrow ((\neg(x < 0)) \land (x^2 = 1)) \right).$$