On Localization Problem in Anchoritic Sensor Networks *

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We introduce a new class of anchoritic *sensor networks, where the chatter between nodes is undesirable or infeasible. We approach the localization problem in such networks by analyzing the correlations of environmental measurements (modeled as random fields) at pairs of nodes.*

I. Introduction

The "coordinate-free" localization (also known as positioning or location) problem in sensor network has attracted significant attention in the literature, see, e.g. a survey [6]. The problem is to determine positions of the nodes in the network without using some absolute reference information, like GPS or direction/distance information relative to some known system of beacons (in other words, when no global system of references is present). Coordinate-free localization problem is therefore to determine the *absolute* positions of the nodes (i.e. *global data*) using only the *local data*, for example the internode distances or directions data. This local-to-global localization problem presents a serious research challenge and the amount of work on it is rapidly growing, see for example [13, 7, 4, 14, 17, 2, 5, 9, 11, 16, 15, 8, 1, 10]. In what follows, we will restrict our attention to this kind of localization problems.

In majority if not all publications addressing the localization problem, gathering the local data assumes extensive *internode communications*, that is a bidirectional exchange of signals between (at least close enough) nodes. Typically one considers a combination of RSS(received signal strength), ToA (time of arrival) and AoA (angle of arrival) data that are used, perhaps in a distributed fashion, to reconstruct the mutual positions of the nodes, and therefore their positions on the plane (up to a Euclidean motion, or uniquely, if some beacons with known positions are present).

It is useful to decouple the global position reconstruction problem into two steps: the first stage is *to recover the internode mutual positions data*, and the second stage is to use these data for *the reconstruction of the global positions of the nodes*, again, perhaps up to an isometry.

In this note we address only a variant of the first step, the recovery of the distance data.

In the literature on the localization problem, the implicit assumption that the relative position information should be gained from the signal exchange (*chatter*) between the nodes and the subsequent processing of the characteristics of this exchange seems necessary. Yet, as we argue below, the requirement that the sensors are able to regularly emit and process information is undesirable in many applied situations. Assuming this for an instant, the question arises:

> *Can the localization problem, or rather the first stage of the localization problem, recovery of the internode distance data, be solved under condition that the sensor nodes do* not *have the ability to* exchange *signals?*

(Clearly, the sensors have to be able to *gather* some signals and eventually to *transfer* them to a processing unit — a sensor which is unable to do even that much can be removed without any detrimental effect for the network operation).

We will refer to such deprived of ability to chat sensors as **anchoritic** *sensors.*

It is perhaps counterintuitive that the localization problem for anchoritic sensor networks can be solved. *Yet we argue that this is the case if we can exploit the correlation structure in the environmental measurements (these measurements can be either the primary data which the sensors aimed to collecting, or some auxiliary background fields).*

This is the main contribution of this note. Before presenting our case, we need, however, to give answer to the following natural questions:

- When and where anchoritic sensor networks are necessary?
- If the sensors do not communicate, how the measurements they are collecting are transfered to the center?

*Jian Tan was summer intern at Bell Laboratories while working on this project.

These questions are addressed in the next subsection. In the last subsection of the introduction we describe the essentials of our approach to anchoritic sensor network localization problem. The rest of the note is organized as follows: after a brief review of the related literature, we set the stage in section III and give an exposition of our approach to the localization problem. Further, we describe (in section IV) several models of the environmental random fields which can be used to recover the mutual distance data in anchoritic sensor networks. Section V presents the results of our numeric simulations, which are followed by the conclusion.

We do not present any proofs of formulated theorems, as they are rather immediate on one hand and extraneous relative to our main contribution. A preprint with the proofs is available on request.

I.A. Motivation

Anchorite sensors

Situations where the sensors collecting the data are anchoritic (i.e. can not exchange signals) are much more widespread than one might suppose.

First of all, if the number of sensors grows to 10^5-10^7, the cost of any extra feature with which the sensors are provided scales correspondingly, and adding even the least expensive CPU or upgraded battery can make the project prohibitively expensive. Requiring the ability to listen to the neighbors' signals and to analyze them can increase the cost of a single devise by an innocuous amount of tens of dollars, which translates into tens of millions of dollars of cost increase for a large sensor network. Hence, the idea of a *bare-boned* sensor dust might force the developers to adopt the *anchoritic* requirements.

Another obvious situation when the sensor chatter is undesirable is the battlefield or other applications involving an adversary. Transmitting signals during the mission reveals the presence of the sensors, and therefore makes them vulnerable either to suppression or to manipulation.

Yet another situation when the chatter between the sensor network nodes can be infeasible arises in the networks immersed into an environment where the signal propagation is either weak or too noisy to infer reliably the internode mutual position informations. For example, the sensors deployed on the seabed would have difficulties using the radio signals to communicate.

There are further applications where sensor networks should be dumb and mute, but in general the mentioned above causes — cost considerations, requirement of silence during information gathering stage, physical obstacles to chatter — seem to cover most of these situations.

Information delivery

Now, if sensors in the network are silent, how they are supposed to transfer (reveal) the data to their owner? There are again several scenarios.

First, the sensors (or their data storage units) can be physically collected after their mission is completed. An important example where this approach can be used involves sensor networks deployed at the seabed. Buoyant sensors can be attached to heavy ballasts to sink them. After some time interval during which the sensors perform their measurements, the ballast is released and the sensors emerge to the surface where they can be collected to process the information. (Clearly, the original positions of the sensors cannot be reliably estimated just by their locations on the sea surface.)

Another way to communicate the collected data is relevant for sensor networks with very low energy budget, or sensor network which should not reveal itself until the data are collected. Here, the sensors are provided with a radio able *to send* signals. However, for the reasons stated above, the communication should be one-time and short. Hence, the sensors transmit just once during their operation cycle, transferring to the center all the information they gathered. After that, either the sensors are compromised, or have no more energy to transmit.

In either case, the center should be able to recover the original positions of the sensors basing only on the information available, that is the individual measurements by each sensor, which is oblivious of the positions or the existence of other sensors in the network.

I.B. Our approach

We approach the problem of recovering the mutual displacements or distances between the nodes in an anchoritic sensor network by exploiting the time-space correlation structure of some random field observed by the sensors. This random field (or several fields) can be the primary data which the sensors are tasked with collecting, or some auxiliary measurements performed with the sole purpose to solve the localization problem. The crucial intuitive idea behind this approach is

The correlations between measurements at different sensors decrease with the distance between the sensors.

More precisely, we assume that a field ξ can be measured by the sensors $\mathbf{N} = \{1, 2, \ldots, N\}$ in the network at (synchronized) instants, say $t_o, t_1, \ldots, t_K$. For a subset $I = \{i_1, i_2, \ldots, i_S\} \subset \mathbf{N}$ one can then form *empirical instantaneous correlation functions*

$$k(I) = k^{-1} \sum_{k=1}^{K} \xi_{i_1}(t_k) \cdot \ldots \cdot \xi_{i_S}(t_k)$$

where $\xi_i(t_k)$ are the measurements of the field ξ by the sensor i at instant t_k.

If we denote the position of the sensor i located as $z_i = (x_i, y_i)$[1], the intuitive picture is that the empirical correlation function, in the limit of $k \to \infty$, converges to its expected value,

$$\kappa(I) = \mathbb{E}\xi_{i_1}(t) \cdot \ldots \cdot \xi_{i_S}(t),$$

at least when the field ξ is stationary and ergodically depends on time. In many cases, the correlation function clusters, that is

$$\kappa(I) \approx \prod_{i_s \in I} \kappa(i_s)$$

as the pairwise distances between the points in I increase. If, in addition, the correlation functions are continuous, the cumulants (given formally as logarithm of the exponential generation function for correlation function,

$$\sum_l \frac{c_l s^l}{l!} = \log(\sum_l \frac{\kappa_l s^l}{l!}),$$

where $\kappa_l = \kappa(\{1, \ldots, l\}))$ are nonvanishing at the diagonal (that is for the sets of points spatially close) and vanish far from the diagonal (that is where the pairwise distances between the points grow), see [12].

We will concentrate in this note on the pairwise cumulants, which reduce to the standard 2-point correlation

$$c = c_2(i, j) = \mathbb{E}\xi_i\xi_j - \mathbb{E}\xi_i\mathbb{E}\xi_j,$$

(remark that $c(i, i) = \mathbb{V}(\xi_i) > 0$ and also $c(i, i) > c(i, j), j \neq i)$[2], so that if c is continuous, $c(z, z') > 0$ for all z, z' close enough.

[1] We concentrate on the plane localization problem in this note; our approach can be used in any dimension or more generally, in an abstract metric space. For example, the sensors can be viruses planted at unknown nodes of a virtual networks unseen from behind a firewall. We defer the discussion of this generalization to a separate publication.

[2] We are leaving generalizations to higher cumulants for later work (see however section VI.

The cumulants described above capture the dependence between the *instantaneous* values of random fields. One can of course exploit also the dependence between different time values at different points. A general way to do so is to extend the range of the random field ξ, considering the new field $\tilde{\xi}_\Delta$ whose value at point $z \in \mathbb{R}^2$ at time t is the trajectory of ξ at (z, t) of length Δ:

$$\tilde{\xi}(z, t) = \{\xi(z, s)\}_{s \in [t, t+\Delta]}.$$

The 2-point correlation functions involve in this case some general kernel functions $K(s, s')$ and are given by

$$\langle \tilde{\xi}, \tilde{\xi}' \rangle = \int_{[o,\Delta]^2} \xi(s) K(s, s') \xi(s') ds ds'.$$

(We will use this kind of dependences in one of our examples.)

In subsequent sections we will present several examples where the values of cumulants, and of the empirical cumulants, can serve as an efficient proxy for the internode distances, thus effectively resolving the first part of the localization problem for the anchorite sensor networks.

II. Related Work

Coordinate-free positioning from proximity data

As we mentioned above, most of the publications on the "coordinate-free" localization problem follow the path of position reconstruction from the proximity data. These approaches typically assume the distances being given by RSS data resulting from the internode chatter, or just by the connectivity patters of the ad-hoc network formed by the nodes. After that some geomteric considerations are used, followed possibly by iterative adjustment and fine tuning. The works following to some degree this pattern are [14, 17, 2, 5, 9, 11, 16, 8, 1, 10].

II.A. Using correlations

The correlations between the measurements has been used in the sensor (or ad-hoc) networking context, most notably to develop coding schemes exploiting them. One has proposed to use correlation in the measurements to improve the netowrk throughput. Similarly, using correlated signals in ad-hoc wireless MIMO networks can be seen to improve the transmission rates.

Yet, our approach appears to be new.

III. Setup

We assume rather standard setting: in a plane region $A \subset \mathbb{R}^2$, N points are selected independently and uniformly with respect to Lebesgue measure. We will assume throughout the note that the area of A is normalized to the unity.

The positions of (say, first) $B < N$ points (called local *beacons*) of the sample are known. The positions of the rest of the sample are unknown, and are to be determined.

We denote the plane coordinates of a point as $z = (x, y)$ so that i-th point of the sample is

$$z_i = (x_i, y_i).$$

If the mutual distances $d_{ij} = |z_i - z_j|$ are known, and the points of the sample $\mathcal{Z} = \{z_1, \ldots, z_N\}$ are in general position (which happens almost surely in our model), the whole configuration $\mathcal{Z}$ can be reconstructed up to an isometry of the plane preserving the positions of beacons (that is up to a rotation if $k = 1$ or up to a axis symmetry of $k = 2$). In fact, it is evident that for $B \geq 3$, the knowledge of the distances to the beacons is enough to reconstruct the position of a node.

III.A. Proximity graph

In an anchoritic network, the sensor do not know their mutual positions, and we resort to the measurements of (empirical) cumulants. Assume that a random ergodic field ξ is present, and that the pairwise cumulants $c(z_i, z_j)$ can be approximated. If a form of law of large numbers is valid, with sample size S large enough the cumulants $c(z_i, z_j)$ can be approximated with desired precision for all pairs (i, j) by the empirical cumulants.

To construct the proximity graph $\Gamma = \Gamma_N$ we connect each node i to k_N nodes having *largest* values of the (empirical) cumulant. For an ergodic field such that

$$c(z, z) > c(z, z'), z \neq z',$$

and a sequence $k_N = o(N)$, the resulting graphs Γ_N would approximate corresponding k_N-nearest neighbor graph based on the Euclidean distances between nodes [3]. In fact, the following result is valid:

Theorem 1 *Let $k_N = \log N^c, c > 1$. For any $\epsilon > o$ there exists $N(\epsilon)$ such that for $N > N(\epsilon)$, with probability at least $1 - \epsilon$,*

$$\left| h_{ib}^{(N)} \sqrt{\frac{k_N}{\pi N}} - d_{ib} \right| < \epsilon$$

for any $b = 1 \ldots, B, i = B + 1, \ldots, N$. Here $h_{ij}^{(N)}$ is the distance between nodes i and j in Γ_N and d_{ij} is the Euclidean distance between z_i and z_j.

In other words, knowing the cumulants is enough to reconstruct, with arbitrary precision, the positions of all nodes in the networks, assuming that the network is large enough.

In the next section we consider several models for various 'real-life" inspired ergodic random fields which could be used for the distance estimation purposes.

IV. Models of Random Background Fields

IV.A. Boolean Model

This model imitates an anchorite sensor network with the shadow/light patterns used as the auxiliary random field. The sensors are assumed scattered uniformly in an open space. The shadow/light detection can be done in an extremely inexpensive fashion (both in terms of hardware costs and energy consumption).

To model the shadow patterns generated by clouds, we will apply the widely used *Boolean model* (see e.g. [3]). The model is specified by

- point process $\mathcal{P}$, and
- class of bounded random sets $\mathcal{B}$.

(To keep matters simple, we assume that $\mathcal{P}$ is a Poisson point process, and the $B \in \mathcal{B}$ is a circle with a random radius R.) Given the pair $(\mathcal{P}, \mathcal{B})$, the random set C is

$$C = \bigcup_{Z_\alpha \in \mathcal{P}} (Z_\alpha + B_\alpha),$$

where B_α are *iid* realizations of the sets from $\mathcal{B}$.

At each instant t, the sensors located in the field observe the field

$$\xi(z) = \begin{cases} 1 & \text{if} \quad z \in C \quad \text{and} \\ 0 & \text{otherwise.} \end{cases}$$

For thus constructed random field ξ, the correlation function between any 2 points z, z' can be readily found. Let $\mathbb{P}(d\beta)$ be the probability measure on the shape space $\mathcal{B}$. For any $\beta \in \mathcal{B}$ and $z \in \mathbb{R}^2$ define

$$\psi(z) = |\beta \cap (\beta + z)|$$

[3]Recall that a k-NN graph is build on a discrete subset of a metric space in such a way that each vertex is connected to k nearest vertices.

to be the Lebesgue measure of the intersection of $shape$ and its displacement by z. For example, if $shape$ is a ball of radius r, then

$$\psi(z) = 2\left(r \arccos \frac{|z|}{2r} - \frac{|z|\sqrt{4r^2 - |z|^2}}{4} \right).$$

Then one has

Proposition 1

$$c(z, z') = \mathbb{E}\xi(z)\mathbb{E}\xi(z') \left(\int_{\mathcal{B}} e^{-\psi(z-z')}\mathbb{P}(d\beta) - 1 \right)$$

and

$$\mathbb{E}\xi(z) = \int_{\mathcal{B}} e^{-|\beta|}\mathbb{P}(d\beta).$$

If the measurements instants are spaced far enough, one can model the realizations of the random sets $C_s, s = 1, \ldots, S$ as *iid*, and the empirical correlation function between any two points z, z' converges to $c(z, z')$. Applying formulae of Proposition 1 shows that one can recover the internode distance data.

IV.B. Large Clouds

A variant of the Boolean model deals with the unbounded shapes, the "large clouds". We consider here the variant of the model where the clouds are represented as the parallel strips of random widths. More precisely, we consider the random set C to be bounded by a family of parallel lines, which are orthogonal to a direction is chosen uniformly from the unit circle, and whose orthogonal projections to this direction form a Poisson point process of some constant intensity (one can check that this definition is independent of the choice of the origin in the plane).

In other words, $z \in C$ if

$$x_{2i} \leq \langle z, e \rangle \leq x_{2i+1}, i = \ldots, -1, 0, 1, \ldots,$$

where $\langle , \rangle$ denote Euclidean scalar product, e is a random vector chosen uniformly from $\{|e| = 1\}$ and $\{x_k\}_{-\infty}^{\infty}$ is a Poisson point process with constant intensity λ.

Again, in this situation it is easy to find the 2-point cumulant. In fact, if a random set is translation and rotation invariant, and each realization of the set is a union of infinite strips, then

Proposition 2

$$c(z, z') = a - b|z - z'|$$

for some constants $a, b > 0$.

Once again, we see that the cumulant attains its maximum on the diagonal, and therefore can be used for recovering the distance data in anchoritic sensor networks.

IV.C. Random Walkers

This model describes the random field generated by some independently moving objects: as an example, one can imagine sensor registering heat of an animal moving nearby.

To model this, we consider random field represented by a family of random walkers in the area A. At time t a sensor located at position z registers $\xi = 1$ if there is a walker at a distance at most r from z.

We will use the spatio-temporary version of the correlation function, that is we will consider also the lagged correlations

$$\mathbb{E}\xi(z, t)\xi(z', t + \Delta), \Delta > 0$$

This trick increases the cumulant of points close enough to be journeyed between by a random walker over the time Δ, yet further than r away. The precise expression for cumulants in this model is a polynomial in Gaussian functions and is quite cumbersome, so we will not present it here. We just state

Proposition 3 *The 2-point cumulant function for the random walkers model is positive on the diagonal and vainishing at the infinity.*

V. Experimental Results

In this section we present results of simulation experiments on proximity graph reconstruction in anchoritic networks, based on several random fields described above. In all these simulations, N nodes were chosen independently at random from uniform density in the unit square A. These nodes are represented as little squares on the corresponding plots.

V.A. Round clouds — Boolean model

In this simulation, round clouds of random radius uniformly distributed on $[0, .4]$ are modeled as a Poisson random field with density 40. Blue circles on Figure depict a realization of the Boolean model; the empirical cumulants were formed from $S = 1000$ independent samples.

The proximity graph Γ shown on Figure is formed by connecting a given number $(12N)$ of pairs of nodes with the largest value of the empirical cumulant. one can see that this graph strongly resemble a nearest neighbor graph: there are very few edges connecting nodes far away, and all pairs of close nodes are connected.

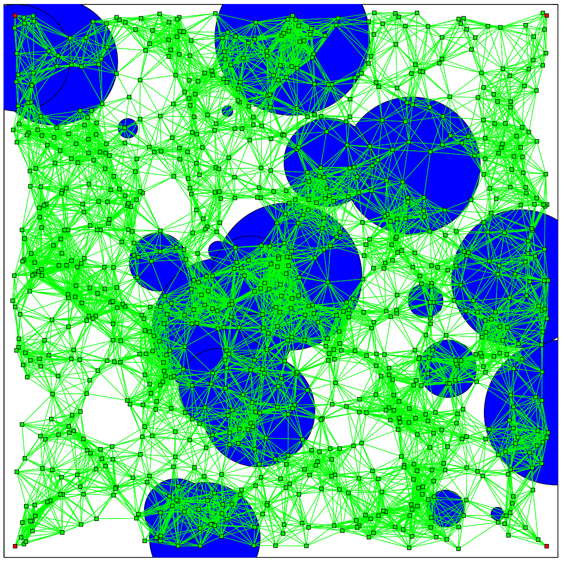

Figure 1: Boolean model with round clouds. Shown is the proximity graph Γ_N for $N = 1000$ nodes.

V.B. Big clouds

The big clouds in this simulation were modeled by half planes (a realization is shown in blue on Figure 2) bounded by lines with isotropic orientation. Again, the pairs of nodes with highest cumulant values were selected; a visual inspection indicates high similarity of this graph with the nearest neighboring graph.

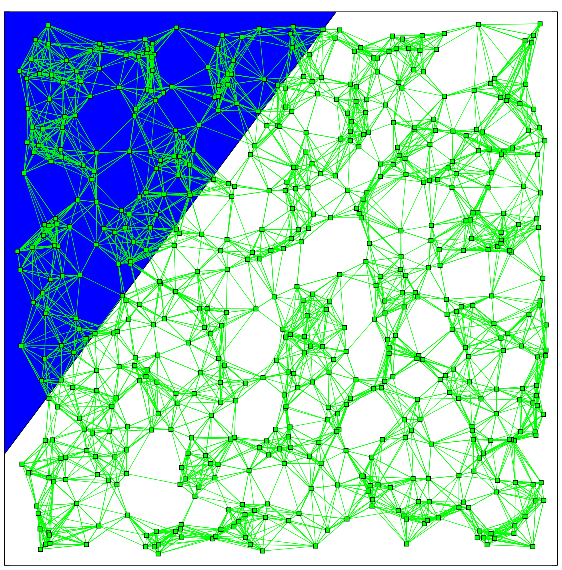

Figure 2: Random sets are formed by isotropic half-planes. Proximity graph for $N = 600$ is shown.

V.C. Random walkers

We consider here $W = 10$ random walkers, which are sensed by a node at distance $r = .13$. Yellow trajectories show the traces of the walkers.

We consider the lagged correlations, with lags equal to $\Delta = 1$ and $\Delta = 2$. For each *node* we then select 6 neighbors with the best values of empirical cumulant.

One can see on Figure 3 that the quality of the proximity graph in situation is worse than that in the previous two examples. The reason is the relatively slow mixing of the random fields defined by the walkers: to ensure the convergence of the empirical cumulants to their average values one needs at least the convergence of the (normalized) occupation measure to the uniform measure. It is evident that the occupation measure in the simulated example is still far away from the uniform measure. Yet, even given this source of imperfection, the result can be reliably used to estimate the proximity of the nodes.

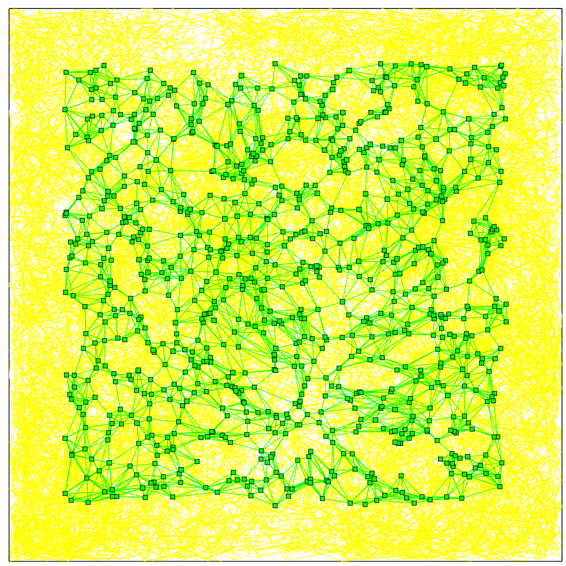

Figure 3: Traces of $W = 10$ random walkers (reflected at the boundary) are shown in yellow. The region in which walkers move is larger than the region A of sensor network deployment, to avoid irregularities at the boundary. Proximity graph for $N = 900$ nodes is shown. One can see several anomalies, like the "panhandle"-shaped hole region near the upper left edge of A, where close nodes are not connected in Γ.

V.D. Quality of distance approximation

A better feeling of the quality of distance data recovery using the correlation between nodes can be gained from the scatter plots, showing the cumulant values versus the distances for nodes pairs. The first two scatter plots (for the round clouds, Figure 4 and large clouds model, Figure 5) show the results for *all pairs* in the sample:

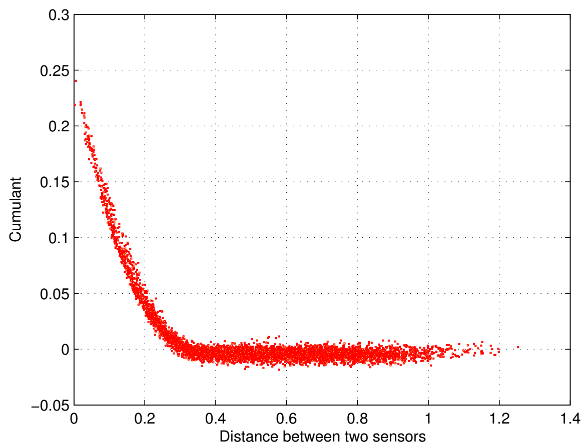

Figure 4: Cumulant-distance scatter plot for the round clouds model. All pairs of points are shown.

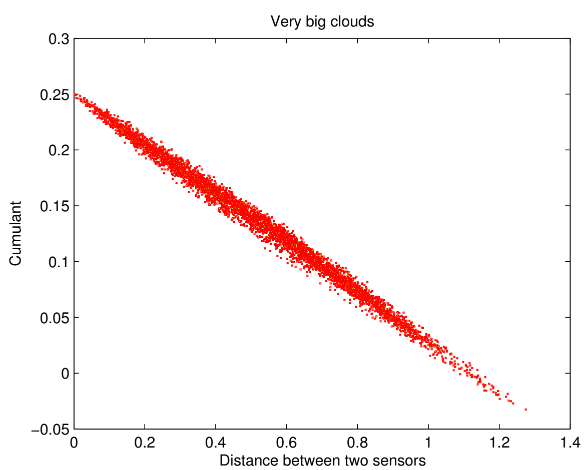

Figure 5: Cumulant-distance scatter plot for the big clouds model. All pairs of points are shown.

For the random walkers model, the scatter plot (Figure 6) shows the cumulant-distance pairs with one of the nodes fixed. The heterogeneity of the occupation measure leads to significantly different ranges of the cumulant at different parts of the region. However, for any particular node, the cumulant can be very efficiently used for selecting the closest nodes, as the plot shows.

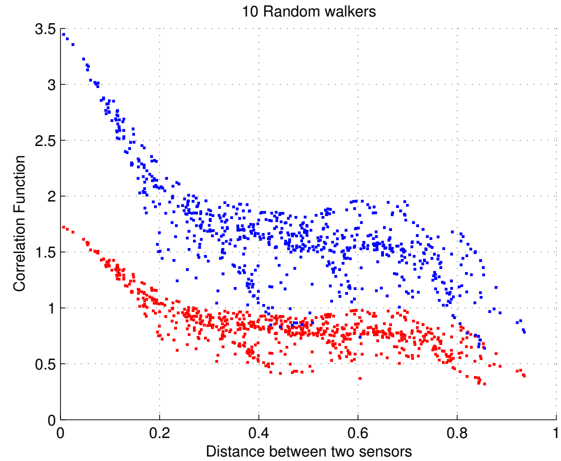

Figure 6: Scatter plot of cumulant versus distance for all pairs $(i, \cdot)$: blue dots show the data for a node i in the region of high occupation density; red dots correspond to a node with low occupation density. The plots are visibly similar, exhibiting high reliability of cumulant estimator.

V.E. End-to-end localization in anchoritic sensor networks

For the round clouds model, we used the cumulant-based proximity graph which was then used to approximate internode distances and ultimately the positions of the nodes. The final steps were done in a rather naive way and did not use any sophisticated machinery. The results are shown below on Figure 7. One can see that boundary effects (caused by the inefficient (distance data)→(position) algorithm used) are rather significant, yet in the interior of the area the positions are recovered quite well.

VI. Conclusion

As demonstrated above, the cumulant evaluation techniques allows one to address the localization problem in anchoritic sensor networks, where the chatter between the nodes is undesirable or infeasible. We would like to indicate several research directions prompted by the present study, which we hope to pursue:

- We concentrated in this study on pairwise cumulants. This fits into the customary context of the existing approached to the localization problem in restricting attention to pairwise mutual positions of the nodes. Yet, the study of correlations allow one to gain much more information by considering the higher order cumulants. For example, in the big clouds models, the leading non-constant term in the k-order cumulant is propor-

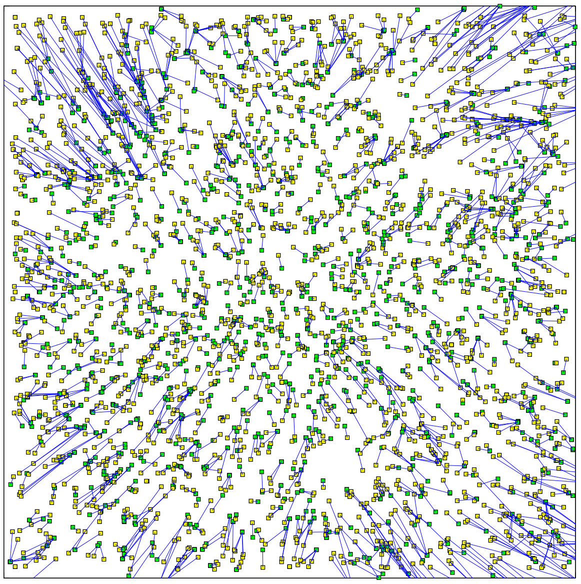

Figure 7: Results of the end-to-end positions estimation in a anchoritic sensor network. Round clouds cumulants were used to generate the proximity graph. The actual node positions are shown as yellow squares connected to their evaluated positions (green squares).

tional to the circumference of the convex hull of the nodes for which the cumulant is computed. Finding systematic ways to use this information is a challenging and promising task.

- We approach the second step of the localization problem — finding the positions from the cumulant data — by generating the proximity graph. More holistic approach would be to generate a Gibbs measure, the ensemble of N nodes in A whose distribution would be consisted with the empirical measurements. Sampling from this distribution would give the most probable positions of the nodes in the region A.

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