Name

1. Suppose $f(x) = 2x$.

   (a) Clearly graph $f(x)$.

   (b) On your graph, show the $x$-values for which $f(x)$ is between 0.5 and 1.5.

   (c) On your graph, show the $x$-values for which $f(x)$ is between 0.75 and 1.25

   (d) For the next questions, give your answers in interval notation.

      i. For which values of $x$ is $f(x)$ between 0.9 and 1.1?

      ii. For which values of $x$ is $f(x)$ between 0.99 and 1.01?

      iii. Let $\varepsilon$ be a small, positive number. For which values of $x$ is $f(x)$ between $1-\varepsilon$ and $1+\varepsilon$?

      iv. We can write this information in a more compact manner. **Note that the difference between two points $a$ and $b$ is given by $|a - b|$**. Fill in the blank.

         Suppose $\varepsilon > 0$. If $|x - 0.5| < \_\_\_\_\_\_$, then $|f(x) - 1| < \varepsilon$
2. Let $f(x) = \sqrt{x}$.

(a) Your boss wants $f(x)$ to be near 10. Specifically, she wants $|f(x) - 10| < 1$. Which values of $x$ will keep her happy?

(b) Let’s turn this into a compact statement again. You can fill in the blank with any positive number that makes the sentence true (but do try to make your answer reasonable). In particular, you are not asked to provide every single value of $x$ for which $|f(x) - 10| < 1$.

If $|x - 100| < _____$, then $|f(x) - 10| < 1$

3. A machinist is required to manufacture a circular metal disk with area 1000cm$^2$.

(a) What radius produces such a disk?

(b) If the machinist is allowed an error tolerance of ±5 cm$^2$ in the area of the disk, what values for the radius of the disk are acceptable?

(c) Let’s rephrase the information again. Feel free to use a decimal that works, even if it cuts out some acceptable values for the radius.

if $|r - _____| < _____$, then $|A - 1000| < 5$

(d) If the machinist is allowed an error tolerance of $\varepsilon$ cm$^2$ in the area of the disk, what values for the radius of the disk are acceptable?

if _____ < $r$ < _____, then $|A - 1000| < \varepsilon$
4. Suppose \( f(x) = 5x - 1 \). Complete the statement.

Let \( \varepsilon > 0 \). If \( |x - \_\_\_\_\_\_\_\_| < \_\_\_\_\_\_\_\_, \) then \( |f(x) - 3| < \varepsilon \)

5. Suppose \( f(x) = \ln(x) \). Complete the statements.

(a) Let \( M > 0 \). If \( x > \_\_\_\_\_\_\_\_ \), then \( f(x) > M \).

(b) Let \( M > 0 \). If \( 0 < x < \_\_\_\_\_\_\_\_ \), then \( f(x) < -M \). (Recall the domain of \( f(x) \) is all positive numbers.)