1. When a thin circular plate of metal is heated in an oven, its radius increases at a rate of 0.01 cm per minute. At what rate is the plate's area increasing when the radius is exactly 80 cm?

(a) Draw a picture of the metal plate. Label the relevant quantities with variable names.

(b) What do you want to find? (Write your answer in math; don’t just repeat the words of the problem.)

(c) What is a formula that relates the relevant variables?

(d) Take the derivative and solve.

\[
\frac{dA}{dt} = \pi \cdot 2r \cdot \frac{dr}{dt}
\]

\[
\frac{dA}{dt} = 2\pi (80)(0.01)
\]

\[
\frac{dA}{dt} = 1.6\pi \text{ cm}^2/\text{min}
\]
2. A water trough is 10 m long and a cross-section has the shape of an isosceles trapezoid that is 30 cm wide at the bottom, 80 cm wide at the top, and has height 50 cm. If the trough is being filled with water at the rate of 0.2 cubic meters per minute, how fast is the water level rising when the water is 30 cm deep?

(a) Here is a picture of the trough. What are the relevant quantities? Label them in the picture.

\[ \frac{dV}{dt} = 0.2 \text{ m}^3/\text{min} \quad V = \text{volume} \]

(b) In math, what do you wish to find? \[ \frac{dw}{dt} \text{ when } w = 30 \text{ cm} \]

(c) What is a formula for the volume of something of this shape? (It may help to know that the area of a trapezoid is \( A = \frac{1}{2} h (b_1 + b_2) \) as pictured.)

\[ V = \text{Area of trapezoid} \times \text{length} \\
= \frac{1}{2} h (b_1 + b_2) \cdot 10 \]

(d) So to relate the relevant quantities of volume and water level, you can write an equation for the volume of water in the trough at any given water level. First draw the trough with water in it (at a level lower than all the way to the top.)

\[ V = \frac{1}{2} w (0.3 + b_2) \cdot 10 \]
(e) You will need to figure out the length of the top of the water (labelled $b_2$ below) in terms of the water level.

\[ b_2 = .3 + 2x \]

Use similar triangles to find $x$

\[ \frac{W}{x} = \frac{.5}{.25} = 2 \]
\[ W = 2x \quad \text{so} \quad b_2 = .3 + W \]

(f) Now write your formula for volume of water in the trough then write it completely in terms of the water level.

\[ V = \frac{1}{2} W (.3 + .3 + W) \cdot 10 \]
\[ = 5W (.6 + W) \]
\[ V = 3W + 5W^2 \]

(g) Take the derivative and solve for the desired quantity.

\[ \frac{dV}{dt} = 3 \frac{dw}{dt} + 10W \cdot \frac{dw}{dt} \]
\[ 0.2 = \left(3 + 10(0.3)\right) \frac{dw}{dt} \]
\[ 0.2 = 6 \frac{dw}{dt} \]
\[ \frac{0.2}{6} = \frac{dw}{dt} \]
\[ \frac{dw}{dt} = \frac{2}{60} = \frac{1}{30} \text{ m/min} \]
Extra problems to work on at home

3. Sands fall from a conveyor belt at a rate of 10 cubic feet per minute onto a conical pile where the radius of the base of the pile is half of the pile's height. How fast is the height growing when the pile is 5 feet high?

\[
\frac{dV}{dt} = 10 \text{ ft}^3/\text{min}
\]

\[
V = \frac{1}{3} \pi r^2 h
\]

\[
= \frac{1}{3} \pi \left(\frac{h}{2}\right)^2 h
\]

\[
= \frac{\pi}{12} h^3
\]

\[
\frac{dV}{dt} = \frac{\pi}{12} \cdot 3h^2 \cdot \frac{dh}{dt}
\]

\[
10 = \frac{\pi}{4} \left(\frac{5}{2}\right)^2 \frac{dh}{dt}
\]

\[
\frac{dh}{dt} = \frac{40}{25\pi} \text{ ft/min}
\]

4. A small balloon is released at a point 40 feet away from an observer, who is on level ground. If the balloon goes straight up at a rate of 10 feet per second, how fast is the distance from the observer to the balloon increasing when the balloon is 30 feet high?

\[
40^2 + h^2 = x^2
\]

\[
0 + 2h \frac{dh}{dt} = 2x \frac{dx}{dt}
\]

\[
2(30)(10) = 2x \frac{dx}{dt}
\]

\[
2(30)(10) = 2(50) \frac{dx}{dt}
\]

\[
\frac{300}{50} = \frac{dx}{dt}
\]

\[
\frac{dx}{dt} = 6 \text{ ft/sec}
\]