Name: Solns

• No calculators allowed.
• Show sufficient work to justify each answer.
• You have 20 minutes for this quiz.

1. (2 points each) Evaluate the following limits

(a) \[ \lim_{{x \to 5}} \frac{1 + \frac{1}{x}}{5 + x} = \lim_{{x \to 5}} \frac{x + 5}{x(5 + x)} = \lim_{{x \to 5}} \frac{1}{5} = \frac{1}{25} \]

(b) \[ \lim_{{t \to 3}} \frac{2 - t}{t - 3} = \lim_{{t \to 3}} \frac{2 - 2}{t - 3} = \lim_{{t \to 3}} \frac{0}{t - 3} = \frac{0}{0} \text{, so } \pm \infty \]

(c) \[ \lim_{{x \to 1}} \frac{x^2 - 1}{x - 1} = \lim_{{x \to 1}} \frac{(x-1)(x+1)}{x-1} = \lim_{{x \to 1}} (x+1) = 2 \]

2. (2 points) Show precisely how the Intermediate Value Theorem is used to prove that the equation \( x^3 - 2x - 3 = 0 \) has at least one real solution.

Since \( 0^3 - 2(0) - 3 < 0 \)
and \( 2^3 - 2(2) - 3 = 1 > 0 \)
we have a continuous function \( f(x) = x^3 - 2x - 3 \) (so continuous on \([0,2]\))
and \( N = 0 \) is between \( f(0) \) and \( f(2) \)
so there is a \( c \) between 0 and 2
so that \( f(c) = 0 \)
that is \( c^3 - 2c - 3 = 0 \).
3. (2 points) Determine the values of $a$ and $b$ so that $f(x)$ is continuous throughout its domain.

\[ f(x) = \begin{cases} 
2x^2 - 8 & \text{for } x < -1 \\
ax + b & \text{for } -1 \leq x \leq 3 \\
16 - 2x & \text{for } x > 3 
\end{cases} \]

\[
\lim_{{x \to -1^-}} f(x) = \lim_{{x \to -1^-}} 2x^2 - 8 = \frac{d}{dx} [2x^2 - 8] = 2(-1)^2 - 8 = -6
\]

\[
\lim_{{x \to -1^+}} f(x) = \lim_{{x \to -1^+}} ax + b = -a + b \quad \text{(also } f(-1) \text{)}
\]

\[
\lim_{{x \to 3^-}} f(x) = 3a + b = f(3)
\]

\[
\lim_{{x \to 3^+}} f(x) = \lim_{{x \to 3}} 16 - 2x = 16 - b = 10
\]

So \[10 = 3a + b \quad \text{and} \quad -6 = -a + b\]

\[\Rightarrow a - 6 = b\]

So \[10 = 3a + a - 6 \]

\[16 = 4a \]

\[4 = a\]

So \[4 - 6 = b\]

\[-2 = b\]

\[a = 4\]

\[b = -2\]