1. Find derivatives of the following functions

(a) \( f(x) = 2^x + 2xe^x \)
\[ f'(x) = 2^x \ln 2 + 2e^x + 2xe^x \]

(b) \( g(x) = \frac{x^5 - 3x}{\log_3 x} \)
\[ g'(x) = \frac{(5x^4 - 3) \log_3 x - x^{4-3}}{(\log_3 x)^2} \]

(c) \( h(x) = \frac{\sqrt{x + 4x^5} + x^\pi}{x} \)
\[ h'(x) = -\frac{1}{2} x^{-3/2} + 16x^3 + (\pi - 1)x^{(\pi-2)} \]

(d) \( y = \sin^2 x \sin x^2 \sin^2 x^2 \)
\[ \frac{dy}{dx} = 2 \sin x \cos x \sin^3 x^2 + 6x \sin^2 x \sin^2 x^2 \cos x \]

(e) \( g(r) = \frac{8 - r}{\ln(r - 4)} \)
\[ g'(r) = \frac{-\ln(r - 4) - (8 - r) \frac{1}{r-4}}{\ln(r - 4)^2} \]

(f) \( p(k) = \sqrt{\sin(\tan \pi x)} \)
As written \( p'(k) = 0 \) as intended,
\[ p'(x) = \frac{\pi}{2} (\sin(\tan(\pi x)))^{-1/2} \cos(\tan(\pi x)) \sec^2(\pi x) \]

2. Find \( f(f'(x)) \) if \( f(x) = x^2 \).
\[ f(f'(x)) = 4x^2 \]

3. Find \( f'(x) \) in terms of \( g'(x) \) if \( f(x) = g(x + g(a)) \) (where you may assume \( a \) is a constant.)
\[ f'(x) = g'(x + g(a)) \]

4. Use implicit differentiation to find \( \frac{dy}{dx} \)
   (a) \( \sin(x - y) = 2x - 2y \)
   \[ \frac{dy}{dx} = 1 \]
(b) \( \sqrt{x + xy} = 1 + x^2 y^2 \)
\[
\frac{dy}{dx} = \frac{1 + y - 4xy^2 \sqrt{x + xy}}{4x^2y \sqrt{x + xy} - x}
\]

(c) \( x^2 + 2xy - y^2 = 2 \)
\[
\frac{dy}{dx} = \frac{x + y}{y - x}
\]

5. Find the value of \( y'' \) at the point where \( x = 0 \) if \( xy + e^y = e \).
\[
y'' = \frac{1}{e^2}
\]

6. Find the tangent line to the graph of the function at the specified input value.

   (a) \( f(x) = x^x \) at \( x = 2 \)
\[
y = 4(\ln 2 + 1)(x - 2) + 4
\]

   (b) \( y = e^t(cosh t - t^2) \) at \( t = 0 \)
\[
y = x + 1
\]

   (c) \( g(x) = \cos x + \cos 2x \) at \( x = \pi \)
\[
y = 0
\]

   (d) \( h(x) = \frac{\sin^{-1}(x)}{x} \) at \( x = 1 \)
   
   As written, does not exist (\( h \) is not differentiable at \( x = 1 \)). Try at \( x = 1/2 \) instead.
\[
y - \pi = (4\sqrt{3} - 2\pi)(x - \frac{1}{2})
\]

7. The half-life of strontium-90 is 28 days. A sample has a mass of 50 mg initially. Find a formula for the mass remaining after \( t \) days. Find the mass remaining after 20 days. How long does it take the sample to decay to a mass of 2 mg?
\[
m(t) = 50\left(\frac{1}{2}\right)^{t/28}
\]
\[
m(20) = 50\left(\frac{1}{2}\right)^{5/7} \text{mg}
\]
\[
t = 28 \log_{1/2}\left(\frac{1}{25}\right)
\]
\[
days or \quad t = \frac{28 \ln 25}{\ln 2} \text{days}
\]

8. (a) State the Mean Value theorem.

   (b) State the Extreme Value theorem.

   (c) State the definition of a critical number.

   In book or class notes

10. Approximate $(1.999)^4$, $\sqrt{99.8}$, and $e^{-0.015}$ using linear approximation.

$$(1.999)^4 \approx 16 - \frac{4}{125}$$

$$\sqrt{99.8} \approx 10 - \frac{1}{100} = 9.99$$

$$e^{-0.015} \approx 1 - \frac{15}{1000} = 0.985$$

11. Evaluate the following limits. Show sufficient justification. An answer of "does not exist" is not sufficient. If the limit is infinite, be sure to state whether it is $\infty$ or $-\infty$.

(a) \( \lim_{x \to 0} \frac{\sin 3x}{x} \) 

(b) \( \lim_{\theta \to 0} \frac{\cos \theta - 1}{\sin \theta} \)

12. Sketch a graph of \( f(x) = \frac{5x^{2/3}}{} - 2x^{5/3} \) (identifying intercepts, asymptotes, intervals of increase/decrease, intervals of concavity, local extrema, and inflection points.)

Increasing \((0, 1)\), decreasing \((-\infty, 0) \cup (1, \infty)\). No asymptotes, \( \lim_{x \to \infty} f(x) = -\infty \) and \( \lim_{x \to -\infty} f(x) = \infty \). local max at \((1, 3)\). not differentiable at 0 but defined at 0, so must be vertical tangent or sharp point. check \( \lim_{x \to 0^+} f'(x) \) and \( \lim_{x \to 0^-} f'(x) \) (it’s a point since one is positive, one is negative.) concave up on \((-\infty, -1/2)\) and concave down on \((-1/2, 0) \cup (0, \infty)\) and \( x = -1/2 \) gives an inflection point.

13. Sketch a graph of \( \sqrt{x^2 + 1} - x \) (identifying intercepts, asymptotes, intervals of increase/decrease, intervals of concavity, local extrema, and inflection points.)

decreasing on \((-\infty, \infty)\) concave up on \((-\infty, \infty)\). HA: \( y = 0 \) but only obeyed on right. \( \lim_{x \to -\infty} f(x) = \infty \). no critical points, no local max/min, no inflection points.

14. Find absolute and local extrema of \( e^x \).

none

15. Find the absolute extrema of the function \( g(x) = \frac{x}{x^2 - 1} \) on the interval \([0, 3]\).

abs max at \((1, 1)\) abs min at \((0, 0)\)

16. Use the Mean Value Theorem to show that there is exactly one solution to the equation \( e^x = 3 - 2x \).

essentially notice that \( e^x + 2 \neq 0 \) but go through all the steps of MVT to see why this is the crucial observation.