• Do not open this test until I say start.
• Turn off all electronic devices and put away all items except a pen/pencil and an eraser.
• No calculators allowed.
• You must show sufficient work to justify each answer.
• Quit working and close this test booklet when I say stop.
1. (4 points each) Find antiderivatives of the following functions

(a) \( f(x) = 4e^x + \frac{1}{5x} \)

\[
F(x) = 4e^x + \frac{1}{5} \ln |x| + C
\]

(b) \( h(k) = \frac{\sqrt{k} + 3k^5 + 1}{k} \) = \( A \sqrt{k} + 3k^4 + \frac{1}{k} \)

\[
H(k) = 2\sqrt{k} + \frac{3}{5} k^5 + \ln |k| + C
\]

(c) \( g'(x) = \frac{1}{x^2 + 1} + \sqrt{x^2} \), with \( g(0) = 3 \)

\[
q(x) = \tan^{-1} x + \frac{3}{5} x^{5/3} + C
\]

\[
g(0) = \tan^{-1} 0 + 0 + C = C = 3
\]

\[
q(x) = \tan^{-1} x + \frac{3}{5} x^{5/3} + 3
\]

2. (8 points) Find \( \lim_{x \to \infty} \left(1 - \frac{2}{x}\right)^x = 1^\infty \)

\[
\ln L = \lim_{x \to \infty} x \ln \left(1 - \frac{2}{x}\right)
\]

\[
= \lim_{x \to \infty} \frac{\ln \left(1 - \frac{2}{x}\right)}{\frac{1}{x}} = \frac{0}{0}
\]

\[
\text{L'Hopital's Rule:}
\]

\[
= \lim_{x \to \infty} \frac{-2}{\left(1 - \frac{2}{x}\right)} = \frac{-2}{1} = -2
\]

\[
\ln L = -2 \implies e^{-2} = L
\]
3. (3 points each) Circle true if the given statement is always true. Otherwise circle false. You do not need to show work. There will be no partial credit on this problem.

(a) The list of indeterminate forms of a limit would include 0, \( \frac{0}{0} \), 1\(^\infty\), 0\(^0\), 0 \cdot \infty, and \( \infty^0 \).

\text{true} or false?

(b) Newton's Method may fail to give closer approximations to the root of a function if the initial approximation \( x_1 \) is such that \( f'(x_1) \) is close to 0.

\text{true} or false?

(c) Piecewise defined functions are not integrable on their domains.

\text{true} or \text{false}?

(d) If \( f \) and \( g \) are continuous and \( f(x) \geq g(x) \) for \( a \leq x \leq b \), then \( \int_a^b f(x) \, dx \geq \int_a^b g(x) \, dx \).

\text{true} or false?

4. (8 points) Use Newton's method to give a second approximation \( x_2 \) to the solution of the equation \( e^{x-3} = -x + 2 \).

\[ f(x) = e^{x-3} + x - 2 \]

\[ f(1) = e^{1-3} - 1 < 0 \]

\[ f(2) = e^{2-3} + 2 > 0 \]

\[ \text{root between 1 and 2, closer to 2} \]

\[ x_1 = 2 \]

\[ x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \]

\[ = 2 - \frac{e^1}{e^1+1} \]
5. (10 points) At which point on the curve $y = 1 + 40x^3 - 3x^5$ does the tangent line have the largest slope?

WANT pts $(x,y)$ on curve $y = 1 + 40x^3 - 3x^5$.

**maximize** slope of tangent line = deriv.

**maximize** $y' = 120x^2 - 15x^4$.

$0 = 240x - 60x^3$

$0 = 60x(4x - x^2)$

$x = 0, \pm 2$

so slope is

<table>
<thead>
<tr>
<th>-2</th>
<th>0</th>
<th>2</th>
<th>x = 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-)</td>
<td>(0)</td>
<td>(+)</td>
<td>(+x)</td>
</tr>
</tbody>
</table>

local maximums occur at $\pm 2 = x$.

Check slope at $\pm 2$:

slope at $x = 2$

$slope(2) = 15(4)(8 - 4)$

$slope(-2) = 15(4)(8 - 4)$

so slope is maximized at $x = \pm 2$.
6. (3 points each) Suppose that $f$ is an even function and $g$ is an odd function which are integrable on the interval $[-4, 4]$. If
\[
\int_{-4}^{4} f(x) \, dx = -2 \quad \text{and} \quad \int_{-4}^{4} g(x) \, dx = 5,
\]
then evaluate the following quantities.

(a) \( \int_{-4}^{4} f(x) \, dx = 2 \)

(b) \( \int_{-4}^{4} g(x) \, dx = 0 \)

(c) \[
\int_{-4}^{4} 2f(x) + 3g(x) \, dx = 2 \int_{-4}^{4} f(x) \, dx + 3 \int_{-4}^{4} g(x) \, dx = 2 (-4) = -8 \]

(d) \( \int_{0}^{4} f(|x|) \, dx = -2 \)

7. (8 points) Fill in the missing information to show that the given definite integral can be expressed as the limit of a Riemann sum. The only variables appearing in your limit should be $n$ and $i$. You do not need to evaluate this limit.

\[
\int_{1}^{5} 2x^5 + x \, dx = \lim_{n \to \infty} \sum_{i=1}^{n} \left[ \frac{1}{n} \cdot \left( 2 \left( 1 + \frac{4i}{n} \right)^5 + \left( 1 + \frac{4i}{n} \right) \right) \right]
\]

\[
\Delta x = \frac{5 - 1}{n} = \frac{4}{n}
\]

\[x_i = 1 + \frac{4i}{n}\]
8. (8 points) Evaluate \( \lim_{n \to \infty} \sum_{i=1}^{n} \left( \frac{1}{4n} - \frac{7i}{2n^2} \right) \)

\[
= \lim_{n \to \infty} \left( \frac{n}{4n} - \frac{7}{2n^2} \sum_{i=1}^{n} i \right) \\
= \lim_{n \to \infty} \left( \frac{n}{4n} - \frac{7}{2n^2} \frac{n(n+1)}{2} \right) \\
= \frac{1}{4} - \frac{7}{4} \\
= \frac{-6}{4} = \boxed{\frac{-3}{2}}
\]

9. (8 points) The graph of \( f(x) \) is given. Let \( g(x) = \int_{2}^{x} f(t) \, dt \).

Place the following in increasing order: \( g(0) \), \( g(2) \), \( f'(3) \), \( f(2) \), \( g(5) \)

\[
g'(0) = \int_{2}^{0} f = -2.5 \\
g(2) = 0 \\
f'(3) = -1 \\
f(2) = 2 \\
g(5) = 2.5 - \frac{1}{2} = 1.5
\]

\[ g(0) < f'(3) < g(2) < g(5) < f(2) \]
10. (14 points) Let \( f(x) = x^4 - 2x^2 - 3. \)

(a) Find the intervals on which \( f \) is increasing or decreasing.
(b) Find the local maximum and local minimum values of \( f \).
(c) Find the intervals of concavity and the inflection points of \( f \).
(d) Sketch a graph of \( f \).

\[
f'(x) = 4x^3 - 4x \\
\text{\( x = 0, \pm 1 \)}
\]

\[
\begin{align*}
\text{\( f''(x) = 12x^2 - 4 \)} \\
\text{\( \quad 0 = 4(3x^2 - 1) \)} \\
\text{\( \quad x = \pm \sqrt{\frac{1}{3}} \)} \\
\text{\( \text{inf\, pts:} \left( \pm \sqrt{\frac{1}{3}}, \frac{32}{9} \right) \)} \\
\text{\( \quad \left( -\sqrt{\frac{1}{3}}, -\frac{32}{9} \right) \)}
\end{align*}
\]

\[
\text{\( f(\sqrt{\frac{1}{3}}) = \left( \frac{1}{3} \right)^2 - 2\left( \frac{1}{3} \right) - 3 = \frac{1}{9} - \frac{2}{3} - 3 = \frac{1}{9} - \frac{6}{9} - \frac{27}{9} = \frac{-32}{9} \)}
\]

\[
\text{x-inter?} \\
\quad (x^2 - 3)(x + 1) = 0 \\
\quad x = \pm \sqrt{3}
\]
Bonus:

(a) (5 points) Express the limit as a definite integral. \( \lim_{n \to \infty} \sum_{k=1}^{n} \frac{k^4}{n^5} \)

\[ = \int_{0}^{1} x^4 \, dx \]

(b) (5 points) For what values of the constants \( a \) and \( b \) is \((1, 3)\) a point of inflection of the curve \( y = ax^3 + bx^2 \)?

\[
\begin{align*}
3a + b &= 3 \\
3a^2 + 2bx &= f' \\
6ax + 2b &= f'' \\
6a + 2b &= 0
\end{align*}
\]

\[
\begin{align*}
a &= -\frac{3}{2} \\
b &= 3 + \frac{3}{2} \\
b &= \frac{9}{2}
\end{align*}
\]