Math 231 Worksheet #7

Instructions. Put the first and last name of everyone in your workgroup at the top of your paper. Everyone is to do their own worksheet but only one from each group is graded with the score shared. Be sure to show your work and explain your reasoning.

1. Evaluate the integral.
   (a) \( \int x \sec^2 x \tan x \, dx \)
   (b) \( \int \sin^3 \theta \sec^4 \theta \, d\theta \)
   (c) \( \int \frac{dx}{(9 - 4x^2)^{\frac{3}{2}}} \)
   (d) \( \int e^{\sqrt{x}} \, dx \)
   (e) \( \int \frac{x^3 + x - 1}{x^2 - 1} \, dx \)
   (f) \( \int_0^t e^s \sin(t-s) \, ds \)

2. Determine whether the integral is convergent or divergent. If convergent, evaluate the integral.
   (a) \( \int_0^\infty \frac{1}{x^2 + 2x + 2} \, dx \)
   (b) \( \int_2^\infty \frac{1}{x \ln x} \, dx \)

3. Bob has a “fast process” to approximate \( \int_a^b f(x) \, dx \) which he calls \( B^b_a(f) \). He knows that if \( |f^{(3)}(x)| \leq K_3 \) for all \( x \) in \([a, b]\) then \( B^b_a(f) \) approximates \( \int_a^b f(x) \, dx \) to an error no more than \( \frac{K_3(b-a)^4}{9} \). Bob wants to use his process to numerically approximate an integral by subdividing the interval into \( n \) equal pieces and applying \( B(f) \) to each of the smaller intervals and then adding up the result. If Bob calls this new approximation \( B^b_n(f) \), what is an upper bound for the error of \( B^b_n(f) \) to approximate \( \int_a^b f(x) \, dx \)? Why?

4. What is the volume obtained by rotating the graph of \( \frac{1}{x} \) from 1 to \( \infty \) about the \( x \)-axis?