Name _________________________________

• Do not open this test until I say start.
• Turn off all electronic devices and put away all items except a pen/pencil and an eraser.
• No calculators allowed.
• You must show sufficient work to justify each answer.
• Quit working and close this test booklet when I say stop.
1. Find the inverse of the following functions

   (a) \( g(x) = \sqrt{3x - 1} + \pi \)  
   \[ A: g^{-1}(x) = \frac{(x-\pi)^2+1}{3} \]

   (b) \( p(x) = \frac{9}{3-10x} - 3 \)  
   \[ A: p^{-1}(x) = -\frac{9}{10(x+3)} + \frac{3}{10} \]

   (c) \( h(x) = \frac{4-x}{2x-1} \)  
   \[ A: h^{-1}(x) = \frac{4+x}{2x+1} \]

2. Let \( f(x) = \sqrt{x-2} + 2 \) and \( g(x) = 2x^3 - 3 \).

   (a) Find \( g^{-1}(1) \)  
   \[ A: \sqrt{\frac{(x-2)^2+5}{2}} \]

   (b) Find \( (f \circ g)^{-1}(x) \)  
   \[ A: \sqrt[3]{2} \]

3. Find the equation of the line

   (a) perpendicular to \( 4x + 3y = 6 \) passing through the point \((1, -1)\)  
   \[ A: y = \frac{3}{4}x - \frac{7}{4} \]

   (b) with undefined slope passing through the point \((\sqrt{2}, \sqrt{3})\)  
   \[ A: x = \sqrt{2} \]

4. Find the domain of the following functions.

   (a) \( f(x) = \sqrt{x+1} + \sqrt{x+7} \)  
   \[ A: [-1, \infty) \]

   (b) \( q(x) = \frac{4}{\sqrt{x+1}} + |x-1| \)  
   \[ A: (-1, \infty) \]

   (c) \( g(x) = \frac{\sqrt{x-4}}{-2x} + \sqrt{\frac{x-3}{x+3}} \)  
   \[ A: (-\infty, -3) \cup (-3, 0) \cup (0, \infty) \]

5. Given that \( f(x) = -x^2 + 2 \), \( g(x) = 3x + 1 \), and \( h(x) = \frac{2}{x - 4} \), find and simplify

   (a) \( (h \circ f)(x) \)  
   \[ A: -\frac{2}{x^2+2} \]

   (b) \( \frac{(g \circ f)(-1)}{(f \circ f)(-2)} \)  
   \[ A: -2 \]

6. After purchasing a mint condition beanie baby for \$5, its value decreases by \$0.15 per year.

   (a) What is the beanie baby’s value 7 years after purchase?  
   \[ A: $3.95 \]

   (b) How many years will it take for this beanie baby’s value to reach $0?  
   \[ A: 34 \text{ years} \]

7. The graph of a function, \( f \), is shown here.

   (a) What is the domain of \( f \)?

   (b) What is the range of \( f \)?

   (c) What is \( f(-1) \)?

   (d) Find all numbers \( x \) such that \( f(x) = -2 \).

   A:a. \([-3, 4]\), b. \((-3, 0]\), c. 0, d. \( x = -3, 1 \)
8. Determine the limit of the sequence

(a) \( b_n = -75 \)  
A: \(-75\)

(b) \( c_n = (-75)^{n-2} \)  
A: does not exist

(c) \( f(n) = \frac{1 - 2n^2}{-n^3 + n + 1} \)  
A:0

(d) \( a_n = -\frac{2n}{\sqrt{n^2 - 10000}} + 2 \)  
A:0

(e) \( a_n = \sqrt{5n^2 + 55001} \)  
A:\(\infty\)

(f) \( f(n) = \frac{1 + n^5}{n^2 - 3n^5} \)  
A:\(-\frac{1}{3}\)

(g) \( g(n) = \frac{\sqrt{9n^6 - 3n^2}}{2n^3 + 7} \)  
A:\(\frac{3}{2}\)

9. Find a formula for the general term \( a_n \) of each sequence, assuming that the pattern of the first few terms continues. Determine whether the sequence is geometric or arithmetic, and if it is, write it in the standard form. Determine the limit of the sequence.

(a) \( \left\{ \frac{1}{2}, \frac{4}{3}, \frac{9}{4}, \frac{16}{5}, \frac{25}{6}, \ldots \right\} \) \( \frac{n^2}{n+1} \) limit=\(\infty\)

(b) \( \{-4, -1, 2, 5, \ldots\} \) \(-4 + 3(n - 1)\) \(\infty\)

(c) \( \{2, \frac{2}{3}, \frac{2}{9}, \frac{2}{27}, \frac{2}{81}, \ldots\} \) \(2\left(-\frac{1}{3}\right)^{n-1}\) 0

(d) \( \{5, 3, \frac{9}{5}, \frac{27}{25}, \frac{81}{125}, \ldots\} \) \(5\left(\frac{3}{5}\right)^{n-1}\) 0

(e) \( \{-8, -3, 2, \ldots\} \) \(-8 + 5(n - 1)\) \(\infty\)

(f) \( \left\{ \frac{16}{27}, \frac{8}{9}, \frac{4}{3}, 2, \ldots \right\} \) \(\frac{16}{27}\left(\frac{3}{2}\right)^{n-1}\) \(\infty\)

(g) \( \left\{ \frac{-1}{2}, \frac{1}{4}, -\frac{1}{6}, \frac{1}{8}, \ldots \right\} \) \(\frac{(-1)^n}{2n}\) 0

(h) \( b_n = 15 - 2(n - 1) \) limit=\(-\infty\)

(i) \( c_n = 4 \) 4

(j) \( d_n = -2\left(\frac{7}{9}\right)^{n-1} \) 0

10. True or False. Provide justification if true, and a counterexample if false: If \( \{a_n\} \) and \( \{b_n\} \) are divergent, then \( \{a_n b_n\} \) is divergent.
A: false

11. A culture of bacteria doubles every 2 hours. If there are 700 bacteria to start, how many will there be after 16 hours?
A: 700(2^8) bacteria

12. Suppose you have a bank account with $150 in it, and every year it earns 4% in interest. If you never take any money out or add any money, how much money will be in the bank account after 4 years?
A: $150(1.04)^3
13. Determine the long run behaviors for the following functions

(a) \( f(x) = -2x^3 + 4000000x + 5000 \) \( \lim_{x \to \infty} f(x) = -\infty \) \( \lim_{n \to -\infty} f(x) = \infty \)

(b) \( q(x) = 3x^4 + 5x^3 + 6x^5 + x^2 + 3x - 9 \) \( \infty \) \( -\infty \)

(c) \( p(x) = (\pi x^3 - 4)(3x + 2)(4x^2 - 5)(2 - x) \) \( -\infty \) \( \infty \)

(d) \( h(x) = (3 - x)(2x - 3)(2x^2 - 1)(\frac{1}{2}x + 5) \) \( -\infty \) \( \infty \)

(e) \( k(x) = 3(\pi)^2 \) \( 3\pi^2 \)

14. For each function, determine the long run behavior, determine the intercepts of the graph of the function, and sketch a rough graph of the function.

(a) \( g(x) = (x + 2)(3x - 1)^2(6 - x) \)

(b) \( f(x) = (x - 6)^2(x^2 - 3)(3 + x) \)

(c) \( p(x) = 25x^3 + 20x^4 + 4x^5 \)

A: a. x-ints \(-2, \frac{1}{3}, 6\), y-int 12,

b. x-ints \(6, \pm\sqrt{3}, -3\), y-int \(-9(36)\),

c. x-ints \(0, -\frac{5}{2}\), y-int 0,
15. Find the rational zeros of the function

(a) \( f(x) = x^4 - 17x^2 + 72 \)  
A: \( \pm 3 \)

(b) \( g(x) = 3x^4 + 40x^3 + 166x^2 + 240x + 63 \)  
A: \(-7, -3, -\frac{1}{3}\)

16. Divide, clearly stating the quotient and remainder: \( 4x^3 - 5x + 3 \div 2x + 1 \).

A: \( 4x^3 - 5x + 3 = (2x + 1)(2x^2 - x - 2) + \frac{5}{2x+1} \)

quotient = \( 2x^2 - x - 2 \) and remainder = 5