Name ________________________________

• Do not open this test until I say start.
• Turn off all electronic devices and put away all items except a pen/pencil and an eraser.
• No calculators allowed.
• You must show sufficient work to justify each answer.
• Quit working and close this test booklet when I say stop.
1. Simplify completely

(a) \[
\frac{\sqrt{32}x^{-5}y^3}{\sqrt{2}x^3y^{-1}z^6}
\]
\[\text{A: } \frac{2y}{x^7}\]

(b) \[(\sqrt{48} + \sqrt{27})^2\]
\[\text{A: 147}\]

(c) \[
\frac{4^{-\frac{3}{2}}}{4^{-\frac{3}{2}} - 9^{-\frac{1}{2}}}
\]
\[\text{A: } -\frac{3}{5}\]

(d) \[
\left(\frac{-2a}{b^2}\right)^{-2} \frac{a^{-3}b^{-1}}{a^5b^0}
\]
\[\text{A: } \frac{b^3}{4a^{10}}\]

2. Solve

(a) \[3 \left| \frac{2x + 5}{2} \right| + 4 \leq 3\]
\[\text{A: no solution}\]

(b) \[4|x - 7| - 13 = -3\]
\[\text{A: } x = \frac{10}{2}, \frac{9}{2}\]

(c) \[12 - |2x + 1| \leq 9\]
\[\text{A: } (-\infty, -2] \cup [1, \infty)\]

(d) \[5 + |3x + 1| - 1 \leq 4\]
\[\text{A: } x = -\frac{1}{3}\]

3. Solve

(a) \[\sqrt{2x^2 - 7} + x = 3\]
\[\text{A: } x = -8, 2\]

(b) \[\sqrt{x - 5} + \sqrt{x + 7} = 6\]
\[\text{A: } x = 9\]

(c) \[\frac{5x}{x - 4} \leq 1\]
\[\text{A: } [-1, 4)\]

(d) \[x + x^\frac{1}{2} = 12\]
\[\text{A: } x = 9\]

(e) \[\frac{1}{x + 2} \leq \frac{1}{x + 6}\]
\[\text{A: } (-6, -2)\]

(f) \[-x^2 + 8x - 16 \leq 0\]
\[\text{A: all real numbers or } (-\infty, \infty)\]

(g) \[\frac{10}{x^2 - 2x} + \frac{4}{x} = \frac{5}{x - 2}\]
\[\text{A: no solution}\]

4. Find the domain of the following functions.

(a) \[f(x) = \sqrt{x + 2} + \sqrt{x + 9}\]
\[\text{A: } [-2, \infty)\]

(b) \[g(x) = \frac{4}{4x - 5}\]
\[\text{A: } (-\infty, \frac{5}{4} \cup \left(\frac{5}{4}, \infty\right)\]

(c) \[g(x) = \frac{3x^2}{\sqrt{5}} + \sqrt{x + 2} + \sqrt{\frac{x - 3}{x}}\]
\[\text{A: } (-\infty, 0) \cup [3, \infty)\]

(d) \[k(x) = \frac{1}{\sqrt{x^4 - 4x^2 + x - 4}}\]
\[\text{A: } (4, \infty)\]

(e) \[p(x) = \frac{x^3 - 1}{\sqrt{x - 4} - \sqrt{5}}\]
\[\text{A: } [4, 9) \cup (9, \infty)\]

(f) \[h(x) = \frac{x^2 - 4}{2x - 1}\]
\[\text{A: } (-\infty, \frac{1}{2}) \cup \left(\frac{1}{2}, \infty\right)\]
5. Given that \( f(x) = -x^2 + 2 \), \( g(x) = 3x + 1 \), and \( h(x) = \frac{2}{x - 4} \), find and simplify

(a) \( f(-2), h(1), \text{ and } g(7) \)
A: \( f(-2) = -2, h(1) = -\frac{2}{3}, g(7) = 22 \)
(b) \( 3h(2) + 2|g(-1)|^2 \)
A: 5
(c) \( (h \circ f)(x) \)
A: don’t worry about it
(d) \( \frac{(g \circ f)(-1)}{(f \circ f)(-2)} \)
A: don’t worry about it
(e) \( \frac{f(x + h) - f(x)}{h} \)
A: \( -2x - h \)
(f) \( \frac{h(x - a) - h(x)}{2a} \)
A: \( \frac{1}{(x - a - 4)(x - 4)} \)

6. True or False: If \((-4, \frac{1}{4})\) is on the graph of an odd function \( g \), then \((-\frac{1}{4}, 4)\) is another point on the graph of \( g \).  
A: False

7. If \( f(x) \) is an even function and \( g(x) = (f(x) - 5)^3 \), is \( g(x) \) even, odd, or neither? (Justify your answer)
A: even, check \( g(-x) \)

8. Find the average rate of change of the following functions on the given intervals
(a) whoops sorry...

9. The graph of a function, \( f \), is shown here.

(a) What is the domain of \( f \)?  
A: \([-3, 4]\)
(b) What is the range of \( f \)?  
A: \((-3, 0]\)
(c) What is \( f(-1) \)?  
A: \( f(-1) = 0 \)
(d) Find all numbers \( x \) such that \( f(x) = -2 \).  
A: \( x = -3, 1 \)

10. A rectangular box without a lid is shown. One rectangular side has perimeter 300 inches. Write the surface area of the box as a function of \( x \) and as a function of \( y \). (Recall: the surface area of this box is the sum of the individual areas of all 5 sides.)

A: \( SA(x) = 450x - x^2, SA(y) = (150 - y)(y + 300) \) square inches (equivalent answers are ok)
11. Wesley is building a rectangular fence for his unusually sized rodents with 400 feet of fencing. They must be kept in separate, equally sized, fenced off rectangles within the large rectangle, as shown. Write the area of the large rectangle as a function of \(x\). Simplify your answer.

\[
A(x) = 3x\left(\frac{200-3x}{2}\right) \text{ square feet}
\]

12. When the reciprocal of a number is subtracted from the number itself, the result is one more than the reciprocal of the number. Find the number(s).

A: the number is 2 or \(-1\).