Name ______________________________________

• Do not open this test until I say start.
• Turn off all electronic devices and put away all items except a pen/pencil and an eraser.
• No calculators allowed.
• You must show sufficient work to justify each answer.
• Quit working and close this test booklet when I say stop.
1. (14 points) Let \( f(x) = \frac{x}{x + 3} \) and \( g(x) = \sqrt{2x - 1} \)

   (a) Find \( f^{-1}(x) \).

   (b) Find \( (f \circ f)(-2) \) \( (f \circ g)(25) \).

2. (6 points) Find the equation of the line perpendicular to \( 3x + 2y = 1 \) passing through the point \((3, -1)\)
3. (8 points) The graph of a function, \( g \), is shown here.

   (a) What is the domain of \( g \)?

   (b) What is the range of \( g \)?

   (c) What is \( g(3) \)?

   (d) Find all numbers \( x \) such that \( g(x) = 0 \).

4. (18 points) Determine the limit of the sequence

   (a) \[ p_n = \frac{3 - 2n^2}{3n^3 - n + 2} \]

   (b) \[ b_n = \left( \frac{-6}{7} \right)^{n+2} \]

   (c) \[ h(n) = \frac{\sqrt[3]{3n + 9n^6}}{n^2 + 71} \]
5. (12 points) Find a formula for the general term $a_n$ of each sequence, assuming that the pattern of the first few terms continues.

(a) $\{12, 9, 6, 3, \ldots\}$

(b) $\left\{ \frac{2}{15}, \frac{1}{5}, \frac{3}{10}, \frac{9}{20}, \ldots \right\}$

6. (6 points) True or False. Provide justification if true, and a counterexample if false: If $\{a_n\}$ and $\{b_n\}$ are divergent and $\left\{ \frac{a_n}{b_n} \right\}$ is defined, then $\left\{ \frac{a_n}{b_n} \right\}$ is divergent.
7. (8 points) A piece of string is cut into six segments whose lengths form a geometric sequence. If the shortest length is 3 cm and the longest is 96 cm, find the length of the string before cutting.

8. (8 points) Let \( p(x) = 2x^2 + 4x + 5 \). Sketch a graph of \( p \), clearly labelling all intercepts and the vertex. Graph paper is provided on the last page, but you are not required to use it.
9. (20 points) Let $f(x) = x^4 + 4x^3 - 10x^2 - 28x - 15$.

(a) Determine the long run behavior of $f$.

(b) List all the possible rational zeros of $f(x)$.

(c) Show that $x = 3$ and $x = -1$ are zeros of $f$, and use this information to completely factor $f$. 

(d) Use your work above to sketch a graph of $f$, clearly labelling all intercepts. Graph paper is provided on the last page, but you are not required to use it.

**Bonus:** (5 points) Find a formula for the general term $f(n)$ of the sequence and determine the limit:

$$\left\{ \frac{25}{3}, \frac{16}{5}, \frac{9}{7}, \frac{4}{9}, \ldots \right\}$$