Saturation Number of Ramsey-Minimal Families

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Graph Saturation

Definitions

Given a forbidden graph $H$, a graph $G$ is $H$-saturated if $H$ is not a subgraph of $G$, but for every $e \in G$, $H$ is a subgraph of $G + e$. 

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Given a forbidden family of graphs $F$, a graph $G$ is $F$-saturated if no member of $F$ is a subgraph of $G$, but for every $e \in G$, some member of $F$ is a subgraph of $G + e$. 

The saturation number $\text{sat}(n; F)$ is the smallest number of edges over all $n$-vertex graphs that are $F$-saturated.
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Definitions

Given “forbidden” graphs $H_1, \ldots, H_k$, and any graph $G$, we write $G \rightarrow (H_1, \ldots, H_k)$ if any $k$ coloring of $E(G)$ contains a monochromatic copy of $H_i$ in color $i$, for some $i$. 

Famous Example: $K_6 \rightarrow (K_3, K_3)$, but $K_5 \not\rightarrow (K_3, K_3)$. 

Definitions

A graph $G$ is $(H_1, \ldots, H_k)$-Ramsey minimal if $G \rightarrow (H_1, \ldots, H_k)$ but for any $e \in E(G)$, $G - e \not\rightarrow (H_1, \ldots, H_k)$. 

Less Famous Example: $K_6$ is $(K_3, K_3)$-Ramsey Minimal. 

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Saturation of Ramsey-Minimal Families  

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\[
\begin{array}{c}
\text{\includegraphics[width=0.5\textwidth]{graph.png}}
\end{array}
\]
Definitions

A graph $G$ is **(H₁, ..., Hₖ)-Ramsey minimal** if $G \to (H₁, ..., Hₖ)$ but for any $e \in E(G)$, $G - e \not\to (H₁, ..., Hₖ)$.

Less Famous Example: $K₆$ is $(K₃, K₃)$-Ramsey Minimal.
A graph $G$ is \textbf{$(H_1, \ldots, H_k)$-Ramsey minimal} if $G \to (H_1, \ldots, H_k)$ but for any $e \in E(G)$, $G - e \not\to (H_1, \ldots, H_k)$.

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A graph $G$ is \textbf{(H\(_1\), \ldots, H\(_k\))-Ramsey minimal} if $G \rightarrow (H\(_1\), \ldots, H\(_k\))$ but for any $e \in E(G)$, $G - e \not\rightarrow (H\(_1\), \ldots, H\(_k\))$.

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\mathcal{R}_{\text{min}}(H\(_1\), \ldots, H\(_k\)) = \mathcal{R}_{\text{min}} = \{ G : G \text{ is } (H\(_1\), \ldots, H\(_k\))-\text{Ramsey minimal} \} \]
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$K_6 \in \mathcal{R}_{\text{min}}(K_3, K_3)$

Definitions

$\mathcal{R}_{\text{min}}(H_1, \ldots, H_k) = \mathcal{R}_{\text{min}} = \{ G : G \text{ is } (H_1, \ldots, H_k)\text{-Ramsey minimal} \}$
Suppose $G$ is $R_{\min}(H_1, \ldots, H_k)$ saturated. $G$ has no subgraph that is $(H_1, \ldots, H_k)$-Ramsey minimal. 

**Pf:** If $G \rightarrow (H_1, \ldots, H_k)$, we delete edges as long as the deletion does not cause an admissible coloring to exist. Adding any edge to $G$ creates a subgraph that is $(H_1, \ldots, H_k)$-Ramsey minimal. For any $e \in E(G)$, $G + e \rightarrow (H_1, \ldots, H_k)$. $G$ is $R_{\min}(H_1, \ldots, H_k)$ saturated iff $G \not\rightarrow (H_1, \ldots, H_k)$ for any $e \in E(G)$. $G + e \rightarrow (H_1, \ldots, H_k)$. 

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Saturation of $\mathcal{R}_{\text{min}}(K_{k_1}, \ldots, K_{k_t})$

**Example**

Let $r := r(k_1, \ldots, k_t)$ be the Ramsey number of $(K_{k_1}, \ldots, K_{k_t})$. Then

$$K_{r-2} \lor \overline{K_s}$$

is $\mathcal{R}_{\text{min}}(K_{k_1} \ldots, K_{k_t})$ saturated.
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**Corollary**

$$\text{sat}(n; \mathcal{R}_{min}(K_{k_1}, \ldots, K_{k_t})) \leq \binom{r-2}{2} + (r - 2)(n - r + 2) \text{ when } n \geq r$$
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**Hanson-Toft Conjecture, 1987**

$$\text{sat}(n; \mathcal{R}_{\min}(K_{k_1}, \ldots, K_{k_t})) = \begin{cases} \binom{n}{2} & n < r \\ \left(\frac{r-2}{2}\right) + (r - 2)(n - r + 2) & n \geq r \end{cases}$$
Hanson-Toft Conjecture

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### Chen, Ferrara, Gould, Magnant, Schmitt; 2011

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sat(n; R_{\min}(K_3, K_3)) = \begin{cases} 
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\[(k_1 + \cdots + k_t - t)K_3 + \overline{K_s} \text{ is } \mathcal{R}_{min}(k_1K_2, \ldots, k_tK_2) \text{ saturated.}\]
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Corollary

\[\text{sat}(n; R_{\text{min}}(k_1K_2 + \cdots + k_tK_2)) \leq 3(k_1 + \cdots + k_t - t)\]
when \(n \geq 3(k_1 + \cdots + k_t - t)\)
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Ferrara, Kim, Y.; 2014

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Construction is generally unique: vertex-disjoint triangles with isolates.
Useful Observation

Ferrara, Kim, Y.; 2014

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Corollary

If $G$ is $R_{\text{min}}(H_1, \ldots, H_k)$ saturated, then $G = G_1 \cup \cdots \cup G_k$, where $G_i$ is $H_i$ saturated and all $G_i$ share the same vertex set.
Thanks for Listening!


