A Ramsey Version of Graph Saturation

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Graph Saturation

Definitions

Given a forbidden graph $H$, a graph $G$ is $H$-saturated if $H$ is not a subgraph of $G$, but for every $e \in G$, $H$ is a subgraph of $G + e$. 

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The saturation number sat($n$; $H$) of a forbidden graph $H$ is the smallest number of edges over all $n$-vertex graphs that are $H$-saturated.

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Given a forbidden family of graphs $F$, a graph $G$ is $F$-saturated if no member of $F$ is a subgraph of $G$, but for every $e \in G$, some member of $F$ is a subgraph of $G + e$.

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Given a forbidden family of graphs $\mathcal{F}$, a graph $G$ is $\mathcal{F}$-saturated if no member of $\mathcal{F}$ is a subgraph of $G$, but for every $e \in G$, some member of $\mathcal{F}$ is a subgraph of $G + e$.
The saturation number $\text{sat}(n;\mathcal{F})$ is the smallest number of edges over all $n$-vertex graphs that are $\mathcal{F}$-saturated.
Definitions

A graph $G$ is $(H_1, \ldots, H_k)$-Ramsey minimal if $G \rightarrow (H_1, \ldots, H_k)$ but for any $e \in E(G)$, $G - e \not\rightarrow (H_1, \ldots, H_k)$. 

Example: $K_6 \rightarrow (K_3, K_3)$, but $K_6 - e \not\rightarrow (K_3, K_3)$. 

$R_{\text{min}}(H_1, \ldots, H_k) = R_{\text{min}} = \{G: G$ is $(H_1, \ldots, H_k)$-Ramsey minimal $\}$
Ramsey-Minimal Families

Definitions

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\[ \text{Definitions} \]

\[ \text{Ramsey minimal} \]

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\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{graph.png}
\caption{Example graph for Ramsey minimal families.}
\end{figure}
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Ferrara-Kim-Yeager (UCD, UIUC)  Ramsey Version of Saturation  24 October 2014
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Saturation of Ramsey-Minimal Families

A graph $G$ is $R_{min}(H_1, \ldots, H_k)$ saturated if and only if:

Example: the graph below is $R_{min}(K_3, K_3)$-saturated.
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Saturation of $\mathcal{R}_{\text{min}}(K_{k_1}, \ldots, K_{k_t})$

**Example**

Let $r := r(k_1, \ldots, k_t)$ be the Ramsey number of $(K_{k_1}, \ldots, K_{k_t})$. Then

$$K_{r-2} \lor \overline{K_s}$$

is $\mathcal{R}_{\text{min}}(K_{k_1}, \ldots, K_{k_t})$ saturated.
Saturation of $\mathcal{R}_{\text{min}}(K_{k_1}, \ldots, K_{k_t})$

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**Corollary**

\[
sat(n; \mathcal{R}_{min}(K_{k_1}, \ldots, K_{k_t})) \leq \binom{r-2}{2} + (r-2)(n-r+2) \text{ when } n \geq r
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Saturation of $\mathcal{R}_{\text{min}}(K_{k_1}, \ldots, K_{k_t})$

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**Hanson-Toft Conjecture, 1987**

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sat(n; \mathcal{R}_{\text{min}}(K_{k_1}, \ldots, K_{k_t})) = \begin{cases} 
\binom{n}{2} & n < r \\
\binom{r-2}{2} + (r - 2)(n - r + 2) & n \geq r
\end{cases}
\]
Hanson-Toft Conjecture

\[
sat(n; R_{\min}(K_{k_1}, \ldots, K_{k_t})) = \begin{cases} 
\frac{n^2}{2} & n < r \\
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\end{cases}
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**Chen, Ferrara, Gould, Magnant, Schmitt; 2011**

\[
sat(n; R_{\text{min}}(K_3, K_3)) = \begin{cases} 
\binom{n}{2} & n < 6 = r \\
4n - 10 & n \geq 56
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Hanson-Toft Conjecture

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Matchings

Example

\[(k_1 + \cdots + k_t - t)K_3 + \overline{K_s} \text{ is } R_{\min}(k_1K_2, \ldots, k_tK_2) \text{ saturated.}\]
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\((k_1 + \cdots + k_t - t)K_3 + \overline{K_s}\) is \(R_{\min}(k_1K_2, \ldots, k_tK_2)\) saturated.

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(5K_2, 5K_2, 5K_2, 5K_2)
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\[
\begin{align*}
\begin{array}{cccc}
\text{Red} & \text{Red} & \text{Red} & \text{Red} \\
\text{Green} & \text{Green} & \text{Green} & \text{Green} \\
\text{Blue} & \text{Blue} & \text{Blue} & \text{Blue} \\
\text{Red} & \text{Red} & \text{Red} & \text{Red} \\
\end{array}
\end{align*}
\]

\[(5K_2, 5K_2, 5K_2, 5K_2)\]
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Useful Observation: “Iterated Recoloring”

Ferrara, Kim, Y.; 2014

Looking (cleverly) at color $i$ allows us to use results from graph saturation of the forbidden subgraph $H_i$. 

Example: Forbidden graphs ($3K_2$, $3K_2$).

- Good coloring makes red-heavy
- Take red subgraph

This (uncolored) subgraph is $3K_2$-saturated.
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good coloring
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Example: Forbidden graphs ($3K_2$, $3K_2$).

![Diagram of a graph with red and blue edges, indicating a good coloring and making red-heavy.]
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Ferrara–Kim–Yeager (UCD, UIUC)
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A diagram showing a graph with nodes colored in red and blue, with a note indicating a good coloring and making it red-heavy.
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![Diagram of a graph with red and blue coloring]

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Example: Forbidden graphs ($3K_2$, $3K_2$).

![Diagram](image)

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![Diagram showing a good coloring and making the red-heavy subgraph $3K_2$-saturated.]

- good coloring
- make red-heavy
- $3K_2$-saturated
Useful Observation: “Iterated Recoloring”

Ferrara, Kim, Y.; 2014

Looking (cleverly) at color $i$ allows us to use results from graph saturation of the forbidden subgraph $H_i$.

Example: Forbidden graphs ($3K_2$, $3K_2$).

![Diagram](image_url)

- good coloring
- make red-heavy
- take red subgraph
Useful Observation: “Iterated Recoloring”

Ferrara, Kim, Y.; 2014

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Example: Forbidden graphs ($3K_2$, $3K_2$).

good coloring
↓
make red-heavy
↓
take red subgraph
Useful Observation: “Iterated Recoloring”

Example: Forbidden graphs (3K₂, 3K₂).

This (uncolored) subgraph is 3K₂-saturated.
Useful Observation: “Iterated Recoloring”

Looking (cleverly) at color $i$ allows us to use results from graph saturation of the forbidden subgraph $H_i$.

Example: Forbidden graphs ($3K_2$, $3K_2$).

This (uncolored) subgraph is $3K_2$-saturated.
Useful Observation: “Iterated Recoloring”

Ferrara, Kim, Y.; 2014

Looking (cleverly) at color $i$ allows us to use results from graph saturation of the forbidden subgraph $H_i$.

Example: Forbidden graphs ($3K_2$, $3K_2$).

![Diagram showing a graph with red and blue edges, and a node marked as uncolored.](image)

This (uncolored) subgraph is $3K_2$-saturated.
Useful Observation: “Iterated Recoloring”

Ferrara, Kim, Y.; 2014
Looking (cleverly) at color $i$ allows us to use results from graph saturation of the forbidden subgraph $H_i$.

Example: Forbidden graphs ($3K_2$, $3K_2$).

This (uncolored) subgraph is $3K_2$-saturated.
Useful Observation: “Iterated Recoloring”

Ferrara, Kim, Y.; 2014

Looking (cleverly) at color $i$ allows us to use results from graph saturation of the forbidden subgraph $H_i$.

Example: Forbidden graphs ($3K_2$, $3K_2$).

![Graph diagram]

This (uncolored) subgraph is $3K_2$-saturated.
Useful Observation: “Iterated Recoloring”

Ferrara, Kim, Y.; 2014

Looking (cleverly) at color $i$ allows us to use results from graph saturation of the forbidden subgraph $H_i$.

Example: Forbidden graphs ($3K_2, 3K_2$).

This (uncolored) subgraph is $3K_2$-saturated.
Thanks for Listening!


