On Vertex-Disjoint Chorded Cycles

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Slides available at
https://faculty.math.illinois.edu/~yager2/research.html
Overview

1 Introduction and Definitions

2 Background

3 Main Theorem

4 Proof Outline

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6 Mixed Cycles

7 Sandia Research
Alexandr Kostochka
University of Illinois

Gexin Yu
College of William and Mary
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Introduction

The Königsberg Bridge Problem

Can you start somewhere, traverse every bridge exactly once, and return to the starting point?
The Königsberg Bridge Problem

Can you start somewhere, traverse every bridge exactly once, and return to the starting point?
A graph $G$ is comprised of a set of vertices $V$ and a set of edges $E$, where the edges are 2-element subsets of $V$. 
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$V(G) = \{s, t, u, v, x, y\}$

$E(G) = \{vs, st, tu, xu, uv, vx, xy\}$

$N(x) = \{u, v, y\}$

(Neighborhood of x)
A graph $G$ is comprised of a set of vertices $V$ and a set of edges $E$, where the edges are 2-element subsets of $V$.

- $d(x) = d(u) = d(v) = 3$,
- $d(y) = 1$,
- $d(s) = d(t) = 2$,

$\delta(G) = 1$ (Minimum Degree)
$\sigma_2(G) = 3$ (Minimum Ore Degree)
A graph $G$ is comprised of a set of vertices $V$ and a set of edges $E$, where the edges are 2-element subsets of $V$.

Cycles: $stuvs$, $vuxv$, $stuxvs$

Chorded Cycle: $stuxvs$ with chord $uv$
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Conjectured by Erdős and Pósa

Theorem (Corrádi, Hajnal 1963)

*Every simple graph* $G$ *on* $|G| \geq 3k$ *vertices with* $\delta(G) \geq 2k$ *contains* $k$ *vertex-disjoint cycles.*
Conjectured by Erdős and Pósa

**Theorem (Corrádi, Hajnal 1963)**

Every simple graph $G$ on $|G| \geq 3k$ vertices with $\delta(G) \geq 2k$ contains $k$ vertex-disjoint cycles.


Every simple graph $G$ on $|G| \geq 3k$ vertices with $\sigma_2(G) \geq 4k - 1$ contains $k$ vertex-disjoint cycles.
Theorem (Corrádi, Hajnal 1963)

Every simple graph $G$ on $|G| \geq 3k$ vertices with $\delta(G) \geq 2k$ contains $k$ vertex-disjoint cycles.

Tightness Example for Corrádi-Hajnal
Background

Theorem (Finkel 2008)

Every graph $G$ on $|G| \geq 4k$ vertices with $\delta(G) \geq 3k$ contains $k$ disjoint chorded cycles.

Theorem (Chiba, Fujita, Gao, Li 2010)

Every graph $G$ on $|G| \geq 4k$ vertices with $\sigma_2(G) \geq 6k - 1$ contains a collection of $k$ disjoint chorded cycles.
Background

Theorem (Molla, Santana, Yeager 2017)

For $k \geq 2$, let $G$ be a graph with $n = |G| \geq 4k$ and $\sigma_2(G) \geq 6k - 2$. Then $G$ does not contain $k$ disjoint chorded cycles if and only if $G \in \{G_1(n, k), G_2(k)\}$, where $G_1(n, k) = K_{3k-1, n-3k+1}$ for $n \geq 6k - 2$ and $G_2(k) = K_{3k-2,3k-2,1}$ for $k \geq 2$. 

\[ G_1(n, 2) \quad G_2(2) \]
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The Main Theorem

Theorem (Kostochka, Y, Yu 2018+ ++ )

Let \( k \geq 2 \), \( G \) be an \( n \)-vertex graph with \( n \geq 4k \) and \( \sigma_2(G) \geq 6k - 3 \). Then \( G \) does not contain \( k \) disjoint chorded cycles if and only if \( G \) is not an exceptional graph.

\[ G_1(n, 2) \]
\[ G_2(2) \]
\[ G_3 \]
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The (Very Simplified) Rules

Choose a collection \( C \) of chorded cycles such that

1. the chorded cycles are as small as possible
2. subject to the preceding, the remainder \( R \) has a longest possible path and is as dense as possible.
Choose a collection $\mathcal{C}$ of chorded cycles such that

1. the number of chorded 4-cycles is maximum,
2. subject to the preceding, the number of $K_4$ is maximum,
3. subject to the preceding, the cycles are small
4. subject to the preceding, the cycles have many edges
5. subject to the preceding, the length of a longest path $P$ in $R := V(G) - V(\mathcal{C})$ is maximum. If $|P| = |R|$, then the number of spanning cycles in $G[R]$ is maximum.
6. subject to the preceding, $|E(G[R])|$ is maximum, and
7. subject to the preceding, $\sum_{v \in R} d_G(v)$ is maximum.
The Rules

1. the number of chorded 4-cycles is maximum,

2. subject to the preceding, the number of $K_4$ is maximum,

3. subject to the preceding, the $k$–tuple $(C_1, \ldots, C_k)$ has $(|V(C_1)|, \ldots, |V(C_k)|)$ least lexicographically,

4. subject to the preceding tuple, $(|E(C_1)|, \ldots, |E(C_k)|)$ is greatest lexicographically,

5. subject to the preceding, the length of a longest path $P$ in $R := V(G) - V(C)$ is maximum. If $|P| = |R|$, then the number of spanning cycles in $G[R]$ is maximum, unless $|R| = 4$ in which case we prefer the paw $G[R] = K_{1,3}^+$ over $G[R] = C_4$.

6. subject to the preceding, $|E(G[R])|$ is maximum, and

7. subject to the preceding, $\sum_{v \in R} d_G(v)$ is maximum.
Cases:

1. $G[R]$ does not have a spanning path

2. $G[R]$ has a spanning path
   
   - and $k \geq 3$
   - and $k = 2$
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An Example

Claim

Given a collection of chorded cycles chosen by our rules, if $G[R]$ does not have a spanning path, then $G[V(P)]$ cannot be a cycle.
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Therefore, $C \neq K_{3,3}$.
Claim

Given a collection of chorded cycles chosen by our rules, if $G[R]$ does not have a spanning path, then $G[V(P)]$ cannot be a cycle.

There exists $x \in V(C)$ such that $N(x) \supseteq \{v_1, v_2\}$. Otherwise, …
Claim

Given a collection of chorded cycles chosen by our rules, if $G[R]$ does not have a spanning path, then $G[V(P)]$ cannot be a cycle.

Otherwise, there exists a smaller cycle, a contradiction.
An Example

Claim

Given a collection of chorded cycles chosen by our rules, if \( G[R] \) does not have a spanning path, then \( G[V(P)] \) cannot be a cycle.

\[ \|\{v_1, v_2\}, C\| \geq 5 \]

If \( w_1 \) and \( w_q \) each have a neighbor in \( C - x \), we can construct 2 disjoint chorded cycles.
An Example

Claim

Given a collection of chorded cycles chosen by our rules, if $G[R]$ does not have a spanning path, then $G[V(P)]$ cannot be a cycle.

Now, there exists a vertex $x' \in V(C) - x$ with $x' \in N(v_1) \cap N(v_2)$ so we can again get 2 disjoint chorded cycles.
Claim

Given a collection of chorded cycles chosen by our rules, if $G[R]$ does not have a spanning path, then $G[V(P)]$ cannot be a cycle.

This proves the claim.
Mixed Cycles

- Current bounds for the given number of cycles and chorded cycles
  - $\delta(G) \geq 2k_{\text{cycle}} + 3k_{\text{chorded}} - 1$ and
  - $\sigma_2(G) \geq 4k_{\text{cycle}} + 6k_{\text{chorded}} - 1$
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Quantifying the Quality of Predicted Synthetic Aperture Radar (SAR) Data

- Shadow Finding & Analysis
- Pixel Statistics Analysis
- Peak Analysis
Quantifying the Quality of Predicted Synthetic Aperture Radar (SAR) Data

- Shadow Finding & Analysis
Quantifying the Quality of Predicted Synthetic Aperture Radar (SAR) Data

- Pixel Statistics Analysis

After considering different methods, I settled on the Anderson-Darling Test

\[ AD = \frac{nm}{n + m} \int_{-\infty}^{\infty} [F(x) - G(x)]^2 \cdot \frac{1}{H(x)(1 - H(x))} dH(x) \]
Quantifying the Quality of Predicted Synthetic Aperture Radar (SAR) Data

- Peak Analysis
Thank You
Some References


