MATH 241 :: Calculus III

Mock Exam I

September 22, 2018

Name: ________________________________

- Show **ALL** your work.
- There are **NO** calculators allowed on this exam.
- This exam has 9 questions.
1. Mark the following as True or False

<table>
<thead>
<tr>
<th>Equation</th>
<th>T or F</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) If the planes $ax + by + cz = d$ and $Ax + By + Cz = D$ are parallel then $a = A$, $b = B$, and $c = C$.</td>
<td></td>
</tr>
<tr>
<td>b) If $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are the standard unit vectors, then $(\mathbf{k} \times \mathbf{i}) \times \mathbf{i} = \mathbf{j} \times (\mathbf{i} \times \mathbf{k})$.</td>
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<tr>
<td>c) Two non-parallel planes with normal vectors $\mathbf{v}$ and $\mathbf{w}$ intersect in a plane with normal vector $\mathbf{v} \times \mathbf{w}$.</td>
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<tr>
<td>d) The volume of the parallelepiped spanned by the vectors $(1, 0, 0), (0, 2, 0)$ and $(0, 0, 3)$ is 6.</td>
<td></td>
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<tr>
<td>e) For all vectors $\mathbf{v}$ and $\mathbf{w}$, the vector $\mathbf{w} \times (\mathbf{w} \times \mathbf{v})$ is perpendicular to $\mathbf{v}$.</td>
<td></td>
</tr>
<tr>
<td>f) The number $</td>
<td>\mathbf{u} \times (\mathbf{v} \times \mathbf{w})</td>
</tr>
<tr>
<td>g) The equation $(\mathbf{u} + \mathbf{v}) \cdot (\mathbf{u} - \mathbf{v}) = 0$ implies $</td>
<td></td>
</tr>
<tr>
<td>h) If $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$ then $\mathbf{b} = \mathbf{c}$.</td>
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</tr>
<tr>
<td>i) Two lines in $\mathbb{R}^3$ are either parallel or intersect.</td>
<td></td>
</tr>
<tr>
<td>j) Two planes in $\mathbb{R}^3$ are either parallel or intersect.</td>
<td></td>
</tr>
</tbody>
</table>

2. Consider the partial differential equation $u_t = u_{xx}$. Determine which of the following are solutions of this equation:

- A. $u(x, t) = e^{-t} \sin(x)$
- B. $u(x, t) = \sin(x - t)$
- C. $u(x, t) = e^{-t} \cos(x)$
- D. $u(x, t) = \cos(x - t)$
- E. $u(x, t) = e^{x+t}$
3. Find the limit, if it exists, or show that the limit does not exist. Show all your work.

(a) \[ \lim_{{(x,y) \to (0,0)}} \frac{x^4 y}{x^8 + y^2} \]

(b) \[ \lim_{{(x,y) \to (0,0)}} \frac{\sin(x^2 + y^2)}{x^2 + y^2} \]

(c) \[ \lim_{{(x,y) \to (1,-1)}} e^{-xy} \cos(x + y) \]
4. Consider the differentiable function $f : \mathbb{R}^2 \to \mathbb{R}$ whose level curves (or contours) are shown in the figure.

(a) What is the sign of $f_x(P)$? Negative Zero Positive.

(b) What is the sign of $f_{yy}(P)$? Negative Zero Positive.

(c) What is the sign of $f_{xy}(P)$? Negative Zero Positive.

(d) Circle the best estimate for $h'(0)$, where $h(t) = f(t^2 + 4t + 2, \cos(t) + 1)$.

\[ -4 \quad -2 \quad 0 \quad 2 \quad 4 \]
5. Consider the following cross-sections:

![Figure 1: x-fixed cross sections](image1)

![Figure 2: y-fixed cross sections](image2)

![Figure 3: z-fixed cross sections](image3)

And consider the following quadric surfaces:

![Figure I](image4)

![Figure II](image5)

![Figure III](image6)

(a) Choose which of the above quadric surfaces (I,II, or III) has the above cross sections.

(b) Match EACH of the following equations to the corresponding surface (I,II,III) shown above:

A. \( z^2 = 1 + 4x^2 + 4y^2 \)
B. \( x^2 + y^2 = z^2 \)
C. \( z = x^2 - y^2 \)
6. Consider the plane $P$ defined by $x + y + z = 1$ and a line $L$ defined by
\[
\frac{x - 3}{a} = \frac{2 - y}{a + 1} = \frac{z + 1}{3 - 2a}.
\]
(a) For what value of $a$ is the line $L$ parallel to the plane $P$?

(b) Find the distance from the line $L$ to the plane $P$ when $L$ is parallel to $P$.

7. Match the functions with their contour plots.

<table>
<thead>
<tr>
<th>function</th>
<th>Enter I,II,III, IV,V, VI here</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x, y) = \sin(x)$</td>
<td></td>
</tr>
<tr>
<td>$f(x, y) = x^2 + 2y^2$</td>
<td></td>
</tr>
<tr>
<td>$f(x, y) =</td>
<td>x</td>
</tr>
<tr>
<td>$f(x, y) = \sin(x) \cos(y)$</td>
<td></td>
</tr>
<tr>
<td>$f(x, y) = xe^{-x^2-y^2}$</td>
<td></td>
</tr>
<tr>
<td>$f(x, y) = x^2/(x^2 + y^2)$</td>
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</tbody>
</table>
8. Find the equation of the plane containing the two lines:

\[ l_1(t) = (-1 + t, 3t, -2 + 2t) \quad l_2(t) = (-3 - 2t, -1 - t, 1 + 3t) \]

9. Consider the vectors \( \mathbf{u}, \mathbf{v}, \mathbf{w}, \) and \( \mathbf{s} \) shown below.

For each of the following, circle the best answer.

(a) A. \( \mathbf{u} = 2(\mathbf{w} + \mathbf{v}) \)  B. \( \mathbf{u} = 2\mathbf{w} - \mathbf{s} \)  C. \( \mathbf{u} = \mathbf{s} - 2\mathbf{w} \)  D. \( \mathbf{u} = 2(\mathbf{s} + \mathbf{v}) \)

(b) A. \( |\mathbf{u} \times \mathbf{v}| > |\mathbf{w} \times \mathbf{u}| \)  B. \( |\mathbf{u} \times \mathbf{v}| = |\mathbf{w} \times \mathbf{u}| \)  C. \( |\mathbf{u} \times \mathbf{v}| < |\mathbf{w} \times \mathbf{u}| \)

(c) A. \( \mathbf{u} \cdot \mathbf{v} > \mathbf{u} \cdot \mathbf{w} \)  B. \( \mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{w} \)  C. \( \mathbf{u} \cdot \mathbf{v} < \mathbf{u} \cdot \mathbf{w} \)

(d) A. \( \text{proj}_s \mathbf{u} = \frac{1}{4}\mathbf{u} \)  B. \( \text{proj}_s \mathbf{u} = 2\mathbf{s} \)  C. \( \text{proj}_s \mathbf{u} = \mathbf{s} \)  D. \( \text{proj}_s \mathbf{u} = \frac{1}{2}\mathbf{u} \)