Proposal Detail:

Proposal Information

Proposal Number: 1702195  
Proposal Title: Beyond Apollonian Circle Packings: Expander Graphs, Number Theory and Geometry  
Received by NSF: 10/14/16  
Principal Investigator: Xin Zhang

This Proposal has been Electronically Signed by the Authorized Organizational Representative (AOR).

NSF Program Information

NSF Division: Division Of Mathematical Sciences  
NSF Program: ALGEBRA,NUMBER THEORY,AND COM  
Program Officer: Matthew Douglass  
PO Telephone: (703) 292-2467  
PO Email: mdouglas@nsf.gov  
Review Information: External Peer Review began on 11/28/16

Proposal Status

Status As of Today Dated: 06/27/17

This proposal has been declined by NSF.

Comments from the cognizant Program Officer:

Dear Xin,

The proposal "Beyond Apollonian Circle Packings: Expander Graphs, Number Theory and Geometry" has not been recommended for funding. This recommendation follows a competition described in a separate Context Statement and in the comments contained in the Review Analysis included below. The reviews and the panel summary obtained in this process are also available below. I ask that you read the reviews, the panel summary, and the review analysis carefully. They should help you in preparing future proposals. If you have questions about this or future NSF proposals, please feel free to contact me.

Matt Douglass  
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DMS-1702195  
Title: Beyond Apollonian Circle Packings: Expander Graphs, Number Theory and Geometry  
PI: Zhang, Xin  
Institution: University of Illinois at Urbana-Champaign

REVIEW PROCEDURE AND OUTCOME: This proposal was reviewed by the Number Theory panel, which was comprised of external experts and was organized at NSF by the Algebra and Number Theory (ANT) program. The panel considered fifty-three proposals submitted to the Division of Mathematical Sciences (DMS) in FY 2017 on various topics associated with number theory and closely related areas. The panel consisted of twelve members whose research specialties represented the scope of the areas considered. It was conducted and observed by ANT
program directors as well as program directors from other programs in DMS.

Each project was assigned to at least three reviewers for review prior to the panel meeting. In some cases additional reviews were sought from other specialists, either via other DMS panels or via mail review. The panel discussed proposals one-by-one with respect to the merit review principles and criteria defined by the National Science Board. Reviews from the assigned panelists formed the starting point of the panel discussion for each project. Other panelists who were familiar with the subject of the proposal also contributed to the panel discussion. Comparisons between proposals took place in these discussions. These comparisons are reflected in the panel summaries.

After the discussion, the panel placed each proposal into one of three categories, labeled Highly Recommended for Funding, Recommended for Funding if Possible, and Not Recommended for Funding. Proposals placed in the Recommended for Funding if Possible category were ranked into equivalence classes of one or more proposals. The panel took into account both the intellectual merit and the broader impacts of each proposal in its ranking. Panelists were instructed to place about 10% of the proposals in the Highly Recommended for Funding category, about 30% in the Recommended for Funding if Possible category, and about 60% in the Not Recommended for Funding category. In the end, the panel placed six proposals in the Highly Recommended for Funding category, seventeen proposals in the Recommended for Funding if Possible category, and thirty proposals in the Not Recommended for Funding category. After the proposals were ranked, a summary of the panel discussion for each proposal, and its ranking, was written and submitted by a scribe (a panelist who was not a reviewer).

This proposal received reviews from three panel reviewers with ratings of Excellent/Very Good, Very Good, and Good. As a result of its deliberations, the panel placed the proposal in the Recommended for Funding if Possible category, where it was ranked eleven/twelve of the seventeen proposals in the category.

REVIEW ANALYSIS: This is a proposal in number theory based on problems that stem from the study of Apollonian circle packings. Various questions are posed and conjectures are stated that center on the main themes of expander graphs, the "affine linear sieve," thin groups and strong approximation, and homogeneous dynamics. The broader impacts of the proposal consist of undergraduate advising.

Reviewers and the panel felt that the intellectual merit of the proposal is generally strong, but with reservations. The reviewer who rated the proposal as Very Good felt that not much progress would be made on part of the proposal and the reviewer who rated the proposal as Good would have liked to have seen a more extensive record of publications by the PI. The panel concluded in the panel summary: "The panel feels that the intellectual merits of this proposal are sound. The PI has a successful track record in the area and the proposed problems are largely doable. The panel felt that the questions on geometric statistics were very interesting, with one panelist commenting that this was the strongest part of the proposal and the key source of novelty. The panel also agreed that Conjecture 2.5 would be interesting, though one panelist felt it could be written more carefully."

Regarding broader impacts, the consensus of the panel is stated concisely in the panel summary: "The panel felt that this was acceptable, but too brief."

The reviewer who rated the proposal as Excellent/Very Good agreed with the panel's evaluation and ranking of the proposal.

RECOMMENDATION: This is a proposal in number theory from an early career researcher. Reviewers and the panel felt that the intellectual merit of the proposal is "sound" and that the broader impacts of the proposal are "acceptable." In the end, the panel felt that some other proposals in this year's exceptionally strong group of proposals made a more compelling case for funding and ranked the proposal in the fourth quintile of those recommended for funding. This ranking places the proposal in the lower two-thirds of the proposals under review. I concur with the panel's evaluation and placement of the proposal. While this proposal is worthy of funding, in the end, the program will only be able to fund at most twenty five percent of the research proposals submitted this year and so regretfully I recommend that the proposal be declined.

J. Matthew Douglass
Program Director
Algebra and Number Theory

Reviews

All of the reviews of your proposal that have been released to you by your NSF program officer can be viewed below. Please note that the Sponsored Project Office (or equivalent) at your organization is NOT given the capability to view your reviews.

**Document:**

**Panel Summary #1**

**Release Date:**

May 18 2017 3:14PM

**Review #1**

**Release Date:**

May 18 2017 2:03PM

**Review #2**

**Release Date:**

May 18 2017 2:02PM
Context Statement

Most research proposals submitted to the Algebra and Number Theory (ANT) program in the Division of Mathematical Sciences are reviewed by one or more of the panels run by the program, with some additional mail reviews. Conference proposals and supplementary requests are reviewed internally, unless the proposed budget exceeds the limit for waiving external reviews. This year, the ANT program expects to recommend support for less than one-third of the competing proposals.

For panel reviews, typically three members of the panel submit independent reviews prior to the panel's discussion of the proposal. Based on the panel discussion a fourth panelist prepares a panel summary that is approved by the panel. Occasionally a proposal is reviewed by more than one panel or by a combination of panel and mail reviews. Copies of all reviews used in the decision process are made available to the PI in FastLane. In reading them, please keep in mind that reviewers are addressing their comments primarily to the NSF, not necessarily to the PI.

Among the things that reviewers look for in a successful proposal are the formulation of a problem and the approach to its solution, clear exposition in the summary and project description, a conveyed sense that the research can be accomplished, and a vision of what the completion of the research might mean. To facilitate the evaluation process, proposals need to be written with the reviewer in mind, so that the importance of the proposed activity can be readily understood in a broad mathematical context.

Based on the written reviews, the review ratings, and the panel summary program directors develop a picture of each proposal's strengths and likely impact. While reviewers' ratings are taken into consideration, the program directors' final determination is not based on a simple average of these ratings. Among other things, they look for reviewers' perspectives on the intellectual merit and broader impacts of the proposed research. The comments of each reviewer are considered in the context of all reviews that reviewer wrote.

The program receives a large number of high-quality proposals each year and recommendations for particular proposals are often difficult to make. Some factors taken into account when making a recommendation include: appropriate balance among subfields and between new investigators and those previously supported; contribution to initiatives and interdisciplinary efforts; effect on education and human resource development; the total amount of funds available; and NSF policies.

The decision to fund a proposal is usually accompanied by high ratings, although sometimes proposals in risky new areas or proposals having potentially transformative ideas are funded even when they are not given the highest marks. Sometimes, revised versions of proposals declined one year are awarded the following year, and often research that has been supported for multiple funding cycles is declined as other, more competitive, proposals enter the competition. Many meritorious proposals will not be recommended for awards. A decision to decline or to award a proposal should not be construed as a statement by either the NSF or the program about the quality of the proposed research.

Conference proposals submitted to the ANT program under solicitation NSF 16-550 with budget requests up to $50,000 are evaluated in accordance with the NSF Proposal and Award Manual, Chapter V, Section B.2, Exceptions to External Merit Review, item n, which allows the program to forego the use of outside reviewers in assessment of proposals for such conferences and workshops. The decision on a recommendation to award or to decline a conference proposal is arrived at by the ANT program directors acting as a committee of the program. Among the criteria used to evaluate the intellectual merit and broader impacts of conference proposals are the following:

* Timeliness of the event with respect to developments in the area(s) of the conference
* Quality of the invited speakers and the scientific program
* Diversity and breadth of participation by individuals and institutions
* Involvement of participants from underrepresented groups and of students and junior investigators
* Overall impact of the conference on the US mathematical community

Conference proposals with budget requests between $50,000 and $100,000 are evaluated using the criteria just cited but following procedures that entail either more substantial internal NSF review, or external review. Conference proposals whose budget requests exceed $100,000 require external review, just as research proposals do.

As is the case for conference proposals, the decision to award or decline a small supplementary proposal is reached by the ANT program directors acting as a committee of the program, in accordance with paragraph V.B.2.f.(3) of the NSF Proposal and Award Manual, which states that certain proposals, such as supplementary requests not exceeding 20% of the original budgets, are exempt from the usual requirements for external review. Supplementary proposals with larger budget requests require external review. Supplements are in most cases awarded for at most one year.

The Program Directors for Algebra and Number Theory in FY 2017 are:
J. Matthew Douglass (mdouglas@nsf.gov)
Timothy J. Hodges (thodges@nsf.gov)
Anatoly Libgober (alibgobe@nsf.gov)
Andrew Pollington (adpollin@nsf.gov)
Panel Summary #1

Proposal Number: 1702195

Panel Summary:

Panel Summary

This proposal is concerned with Apollonian circle packings, expander graphs, thin groups, and super strong approximation.

The panel feels that the intellectual merits of this proposal are sound. The PI has a successful track record in the area and the proposed problems are largely doable. The panel felt that the questions on geometric statistics were very interesting, with one panelist commenting that this was the strongest part of the proposal and the key source of novelty. The panel also agreed that Conjecture 2.5 would be interesting, though one panelist felt it could be written more carefully.

For broader impacts, the PI plans to lead undergraduate research groups on related projects.

The panel felt that this was acceptable, but too brief. Other proposals from investigators in the PI's career stage typically included the organization of some sort of seminar or special session, or the production of accessible lecture notes. The panel feels that this proposal would have benefited from something similar.

In consideration of the above, and the strength of the other proposals, the panel placed the proposal in the Recommended for Funding if Possible category.

This summary was read by the panel, and the panel concurs that it accurately reflects the panel discussion.

Panel Recommendation: Recommended for funding if possible
Review #1

Proposal Number: 1702195  
NSF Program: ALGEBRA, NUMBER THEORY, AND COM  
Principal Investigator: Zhang, Xin  
Proposal Title: Beyond Apollonian Circle Packings: Expander Graphs, Number Theory and Geometry  
Rating: Good

REVIEW:

In the context of the five review elements, please evaluate the strengths and weaknesses of the proposal with respect to intellectual merit.

This is a good proposal. If successful it would advance knowledge within its field in an original way. The proposal is soundly based, the PI has the requisite skills and has adequate resources.

In the context of the five review elements, please evaluate the strengths and weaknesses of the proposal with respect to broader impacts.

The PI plans to involve undergraduates to collect useful numeric evidence for three of PI’s projects. These projects will provide motivated students with good opportunities to gain research experience and should encourage them to continue in mathematics. The results of these projects may well be publishable in research journals.

Please evaluate the strengths and weaknesses of the proposal with respect to any additional solicitation-specific review criteria, if applicable

NA

Summary Statement

This proposal is concerned with a subject, namely Apollonian packing and expanders, which utilizes techniques from number theory, homogeneous dynamics, algebraic group theory and combinatorics. Expander graphs are sparse, yet highly connected graphs which are widely used in computer science as well as mathematics. Cayley graphs formed by congruence families of algebraic groups are of particular interest. Generally it is known that Cayley graphs are expanders if the moduli are restricted to square free integers, but much less is known otherwise. In this proposal the PI makes a conjecture, 2.5, which he proposes to prove, which would provide families of expenders in which the moduli is not necessarily squarefree. This is a nice idea.

The PI also asks a series of interesting questions in connection with which integers are in algebraic group orbits and has adapted a version of the Hardy-Littlewood method to make some progress.

Broader Impact. The PI plans to involve undergraduates to collect useful numeric evidence for three of PI’s projects. These projects will provide motivated students with good opportunities to gain research experience and should encourage them to continue in mathematics. The results of these projects may well be publishable in research journals.

The PI is working in a currently very fashionable area in which there is a good deal of progress by some powerful investigators. The PI is only two years out from his PhD and has only one paper accepted and two submitted. However it does seem that he has the skills necessary to make progress in this competitive area, and I am inclined to recommend support if possible.
Review #2

Proposal Number: 1702195
NSF Program: ALGEBRA, NUMBER THEORY, AND COM
Principal Investigator: Zhang, Xin
Proposal Title: Beyond Apollonian Circle Packings: Expander Graphs, Number Theory and Geometry
Rating: Multiple Rating: (Excellent/Very Good)

REVIEW:

In the context of the five review elements, please evaluate the strengths and weaknesses of the proposal with respect to intellectual merit.

This proposal is related to affine sieve developed by Bourgain-Gamburd-Sarnak. The fundamental theorem of affine sieve (proved by Salehi Golsefidy-Sarnak) provides the existence of a finite saturation number in the most general setting possible, but this result does not give a good bound on this number. To get such a bound, one needs to have a very specific example to work with. And in that case, one has to come up with particular sieve techniques that are appropriate for the question in hand. These techniques are particularly delicate when one wants to prove results towards local-global conjecture. This conjecture roughly asks what numbers are representable as values of a given polynomial over a given finitely generated group. In this proposal, the PI considers particular examples of this conjecture, that are natural extensions of works of Bourgain, Fuchs, Kontorovich, etc.

In the first part of this proposal, the PI is considering certain infinite index subgroups of various Bianchi groups, and their corresponding Apollonian packings. In joint works with Fuchs and Stange, the PI is investigating the local-global conjecture for these groups. This conjecture for the standard Apollonian packing have been extensively studied in the past decade.

Conjecture 2.5 is a special case of super-approximation conjecture. The PI has formulated this conjecture as it was needed in his work and a similar result was proved earlier by Varj.

The PI would like to investigate the local-global conjecture for an integral matrix coefficient for a finitely generated subgroup of SL(2,Z) containing parabolic elements (Question 5).

The PI would like to investigate the local-global conjecture for a (large) subgroup of SL(2,Z[i]) and the quadratic polynomial of the sum of square of norms of the entries (Question 7).

The PI would like to investigate the Riesz s-energy of the centers of circles with curvature at most N in an Apollonian packing (Question 8). In Question 9, the PI would like to study the nearest neighbor spacing statistics.

Question 10 is tightly related to Questions 8 and 9. This is a very nice question which is in the general frame work of going from a finite covolume setting to an infinite covolume setting.

Question 11 is again in the general frame work of going from a finite covolume setting to an infinite covolume setting. This time, the PI would like to investigate a possible extension of a work of Chas and Lalley on the self-intersection number of geodesics.

In the context of the five review elements, please evaluate the strengths and weaknesses of the proposal with respect to broader impacts.

The PI plans to lead three undergrads to work on the computational aspects of this proposal, and along the way learn the needed concepts. The PI will mentor them to publish their findings.
Please evaluate the strengths and weaknesses of the proposal with respect to any additional solicitation-specific review criteria, if applicable.

Summary Statement

The PI proposes a set of original projects. First part of these projects can help us to find out what exactly was needed in the results proved for integral apollonian packings and take the ad hoc techniques out of those proofs.

I particularly like the second part of the proposal, where the PI would like to get some results for thin Kleinian groups to deduce some results in geometric statistics.
In the context of the five review elements, please evaluate the strengths and weaknesses of the proposal with respect to intellectual merit.

The PI intends to work on the number of interesting and challenging projects in the area of thin groups and super(approximation). He has obtained a number of interesting density-one results, extending the methods of Kontorovich and Bourgain (1) from classical Apollonian packing to Apollonian 3-circle packings and (2) in the case of thin subgroups of the modular group with a Hausdorff dimension close to 1 to allow the presence of parabolic elements. He proposes to work on extending these results to (1)certain subgroups of Bianchi group and (2) subgroups of the modular group without restriction on the Hausdorff dimension. The first problem seems quite tractable, the second, in the opinion of this reviewer, would require a fundamentally new idea.

The PI also intends to work on extending Varju's approach to the proof of spectral gap in the case of Apollonian group to the case of thin subgroups of general linear group. In this context, recent work of Golsefidy on Conjecture 2.3 (proving it for fixed powers of square free integers and powers of primes) must be mentioned.

The proposal could greatly benefit from being written more carefully. For example, in the statement of Conjecture 2.5 condition 1 is a consequence of condition 2; G in condition 3 is not defined.

The PI also plans to work on several interesting problems in geometric statistics and self-intersection of geodesics.

In the context of the five review elements, please evaluate the strengths and weaknesses of the proposal with respect to broader impacts.

The PI plans to work with undergraduates to gain numerical evidence for some of the PI's proposed problems.

Please evaluate the strengths and weaknesses of the proposal with respect to any additional solicitation-specific review criteria, if applicable.

Summary Statement

The PI has a good track record of solving problems in the area of thin groups and super approximation and plans to work on a broad and eclectic range of problems in this area. He has a good chance in solving some of them. This proposal ranks in the middle third of those assigned to this reviewer.