Midterm 3 Practice Problems — December 3rd, 2012

Name: ________________________________

• This is a closed-book, closed-notes exam. No electronic aids are allowed.

• Read each question carefully. Proof questions should be written out with all the details. Explain your answers.

• If you need extra room, use the back sides of each page. If you must use extra paper, make sure to write your name on it and attach it to this exam. Do not unstaple or detach pages from this exam.
**Problem 1.** (10 points)

(a) (2 points) Is the cycle graph $C_3$ bipartite?

(b) (2 points) Is the cycle graph $C_4$ bipartite?

(c) (6 points) For which values of $n \geq 3$ is the cycle graph $C_n$ bipartite?
Problem 2. (10 points) The questions below refer to the graph in Figure 1 (found at the end of the exam).

(a) (3 points) Is there a complete matching for A, B, C? (Either exhibit a complete matching, or explain why one does not exist.)

(b) (4 points) Is there a complete matching for D, E, F, G? (Either exhibit a complete matching, or explain why one does not exist.)

(c) (3 points) What are the connected components of the graph?
Problem 3. (20 points) Examine the graphs $G$ and $H$ in Figure 2.

(a) (4 points) Write down the adjacency matrix for $G$ with respect to the ordering $u_1, u_2, \ldots, u_6$.

(b) (4 points) Write down the incidence matrix for $H$ with respect to the ordering $v_1, v_2, \ldots, v_6$ and $e_1, e_2, \ldots, e_7$. 
(c) (4 points) What is $\sum_{i=1}^{5} \deg(v_i)$?

(d) (4 points) Are $G$ and $H$ isomorphic? Explain your answer!

(e) (4 points) What is the number of simple paths from $u_1$ to $u_2$ in $G$?
Problem 4. (20 points) Many of the questions in this problem refer to the graphs $G$ and $H$ in Figure 2.

(a) (2 points) Draw a disconnected graph with 2 vertices.

(b) (2 points) Are there cut vertices in the graph $G$ in Figure 2?

(c) (3 points) Give an minimal edge cut for $H$ in Figure 2.
(d) (3 points) Is there an Euler path in G? Explain your answer.

(e) (2 points) Is there an Euler circuit in G? Explain your answer.
(f) (3 points) Is there a Hamiltonian path in G? Explain your answer.

(g) (5 points) Is there a Hamiltonian circuit in G? Explain your answer.
Problem 5. (10 points)

1. For which values of \( n \) is the complete graph on \( n \) vertices \( K_n \) planar? Explain your answer.

2. For which values of \( m \) and \( n \) is the complete bipartite graph \( K_{m,n} \) planar? Explain your answer.