(1) Warm-ups:
(a) Verbally define with your group the meaning of the terms: horizontal asymptote, limit at infinity, tangent line, derivative.
(b) Give the two different limits that give the derivative of a function \( f(x) \) at \( a \).
(c) Review why the function \( f(x) = |x| \) is not differentiable at \( x = 0 \).

(2) Find the horizontal asymptotes of
\[
\frac{5x^3 + x^2 - 3}{-2x^3 + x}
\]

(3) Find the horizontal asymptotes of
\[
\frac{\sqrt{x^2 - 2}}{x + 5}
\]

(4) Find the horizontal asymptotes of
\[
\frac{e^{8x} + e^{3x} - 5}{4e^{8x} - 6}
\]

(5) Find \( \lim_{x \to -\infty} (x^7 + 5x^4) \).

(6) Find all real numbers \( a, b, \) and \( c \) so that:
\[
f(x) = \frac{ax^2 + cx}{x^2 + bx + a}
\]

has vertical asymptotes \( x = 1 \) and \( x = 3 \), horizontal asymptote \( y = 3 \), and only one zero.

(7) Find the derivative of any line at any value \( x = a \); i.e., find \( f'(a) \) when \( f(x) = mx + b \).

(8) For \( f(x) = \sqrt{x} \), find \( f'(1) \).

(9) As we discussed in class, the tangent line to any function \( f(x) \) at the value \( x = a \) is the line through \((a, f(a))\) with slope the derivative \( f'(a) \). Find the tangent lines to the
following functions at the following points and sketch a graph of both \( f \) and the tangent line:

(a) The tangent line to \( f(x) = 2x + 3 \) at \( x = 0 \).
(b) The tangent line to \( f(x) = \sqrt{x} \) at \( x = 1 \).
(c) The tangent line to \( f(x) = x^2 \) at \( x = 2 \).

10) Boyle’s Law says that the pressure and volume of an ideal gas are inversely proportional. In other words, for a fixed mass of a gas kept at a constant temperature, there is a constant real number \( k \) so that the pressure the gas exerts on its container may be expressed as the function

\[
P(V) = \frac{k}{V}
\]

(a) Suppose \( a \) is a volume in the domain of \( P(V) \). What does \( P'(a) \) represent?
(b) Calculate \( P'(1) \) and \( P'(2) \).
(c) What can you conclude about the relationship between pressure and volume at \( V = 1 \) versus \( V = 2 \)? For instance, where does changing the volume have more of an effect on the pressure?

11) Let

\[
f(x) = \begin{cases} 
\sqrt{1 - x^2} & -1 \leq x \leq 1 \\
-\sqrt{1 - (x-2)^2} & 1 < x \leq 3 
\end{cases}
\]

(a) Graph this function (hint: recall from geometry that the equation \((x-a)^2+(y-b)^2 = 1\) gives the equation to a circle of radius 1 centered at \((a, b)\); so what does the circle of radius 1 centered at \((2, 0)\) look like?)
(b) On your graph, determine what the tangent line to \( f \) at \( x = 1 \) should look like (assuming it exists)
(c) what happens when you try to calculate the derivative of \( f(x) \) at \( x = 1 \) (hint: use 2 one sided limits to accommodate how \( f(x) \) is defined)
(d) Try to interpret the results of parts b and c.