(1) Warm-ups:
(a) Verbally define with your group the meaning of the terms: “\( f(x) \) is continuous at \( a \)”, “\( f(x) \) is continuous on \([a,b]\)”, Intermediate Value Theorem
(b) Explain how the limit laws help imply that the sum/product/quotient/constant multiples of continuous functions are again continuous.
(c) Explain why \( \frac{x^7 + 9 \tan(x)}{\ln(x) + 9x} \) and \( \sin(x^3) \) are both continuous on their domains.
(d) Explain how the Intermediate Value Theorem tells you that a ball that was 1 meter in the air at one time and on the ground at another time must have been 0.5 meters in the air at some intermediate time.

(2) Find all possible combinations of real numbers \( a, b, \) and \( c \) so that the function:

\[
f(x) = \begin{cases} 
  b & x < -1 \\
  ax^2 + bx + c & -1 \leq x \leq 1 \\
  cx^3 + a & 1 < x < 2 \\
  7 & 2 \leq x 
\end{cases}
\]

is continuous.

(3) Find

\[
\lim_{x \to 0} \arcsin \left( \frac{\sqrt{x^2 + 1} - 1}{x^2} \right)
\]

(4) For \( f(x) = x^2 \sin \left( \frac{1}{x} \right) \), find

\[
\lim_{x \to 0} e^{f(x)}
\]

(5) Find

\[
\lim_{x \to 2} \sqrt[3]{\sin \left( \frac{\pi}{2} \cdot \frac{x^2 - 3x + 2}{x - 2} \right)}
\]
(6) Show that \( \sin^2(x) - e/3 \) has a root \( c \) with \( \pi < c < 3\pi/2 \) (hint: remember \( e \approx 2.7 \)).

(7) Show that \( \arccos(x) - \sin(x) \) has a root (hint: \( -\pi/2 < -1 < 0 \) and \( 0 < 1 < \pi/2 \)).

(8) Suppose a triangle \( \Delta \) in the \( xy \)-plane is built from the points \((-1, 0), (1, 0)\), and a point \( P \) somewhere on the circle \( x^2 + y^2 = 1 \). Show that there is a point \( P \) in the first quadrant so that \( \Delta \) has area \( \frac{\pi}{4} \).

(9) Show that, if the limit

\[
\lim_{x \to a} \frac{f(x) - f(a)}{x - a}
\]

exists and is finite, then \( f \) is continuous at \( a \).