(1) Warm-up: for $f(x) = x^2$ and $g(x) = 7x + 2$, give each of the following functions:
   (a) $3f(x) = 3x^2$
   (b) $f(x + 3) = x^2 + 6x + 9$
   (c) $f(3x) = 9x^2$
   (d) $(f \cdot g)(x) = 7x^3 + 2x^2$
   (e) $(f + g)(x) = x^2 + 7x + 2$
   (f) $(f \circ g)(x) = 49x^2 + 28x + 4$
   (g) $(g \circ f)(x) = 7x^2 + 2$

(2) Find two functions $f(x)$ and $g(x)$ so that $h(x) = \sqrt[3]{x^8 + 5}$ satisfies $(f \circ g)(x) = h(x)$. Take $f(x) = \sqrt[3]{x}$ and $g(x) = x^8 + 5$.

(3) Repeat the previous problem, finding two different functions $f(x)$ and $g(x)$ with $(f \circ g)(x) = h(x)$.
   Take $f(x) = \sqrt[3]{x + 5}$ and $g(x) = x^8$.

(4) For two lines $f(x) = mx + a$ and $g(x) = nx + b$, find $g \circ f$. What kind of function is this?

$$g(f(x)) = g(mx + a) = n(mx + a) + b = mnx + na + b$$

So the composite of two lines is again a line with slope the product of the slopes of the original two lines (this will be important later in the semester).

(5) Let $f(x)$ and $g(x)$ be two functions. Then:
   (a) If $f$ and $g$ are both even, are $h(x) = f(x) \cdot g(x)$ and $i(x) = f(x) + g(x)$ even or odd (or possibly neither)?

$$h(-x) = f(-x) \cdot g(-x) = f(x) \cdot g(x) = h(x),$$ so $h$ is even. $i(-x) = f(-x) + g(-x) = f(x) + g(x) = i(x)$, so $i$ is also even.

(b) If $f$ is even and $g$ is odd, are $h(x) = f(x) \cdot g(x)$ and $i(x) = f(x) + g(x)$ even or odd (or possibly neither)?

$$h(-x) = f(-x) \cdot g(-x) = f(x) \cdot (-g(x)) = -(f(x) \cdot g(x)) = -h(x),$$ so $h$ is odd. $i(x)$ needn’t be either even or odd; for instance, for $f(x) = 2$ and $g(x) = x$, $i(x) = x + 2$ is neither even nor odd.

(c) If $f$ and $g$ are both odd, are $h(x) = f(x) \cdot g(x)$ and $i(x) = f(x) + g(x)$ even or odd (or possibly neither)?
\[ h(-x) = f(-x) \cdot g(-x) = (-f(x) \cdot -g(x)) = f(x) \cdot g(x) = h(x), \text{ so } h \text{ is even.} \]
\[ i(-x) = f(-x) + g(-x) = -f(x) - g(x) = -i(x), \text{ so } i(x) \text{ is odd.} \]

(6) Given two functions \( f(x) \) and \( g(x) \), how do I find the domain for \( f \circ g \)? That is, is the domain of this function just the domain of \( g \), or is there something trickier going on? Hint: consider examples like \( f(x) = \sqrt{x} \) and \( g(x) = x^2 + 3 \).

First, since we’re plugging things into \( g \), the domain of \( f \circ g \) is at most the domain of \( g \). But we also need to be careful about what values \( g(x) \) are getting plugged into \( f \). So the domain of \( f \circ g \) is the set of real numbers \( x \) that are in the domain of \( g \) for which \( g(x) \) is in the domain of \( f \).

(7) Graphing variations of \( \sin(x) \):
(a) Explain why \( \sin(x) \) satisfies \(-1 \leq \sin(x) \leq 1 \) for every real number \( x \).

Since \( \sin(\theta) \) gives the \( y \)-coordinate on the circle \( x^2 + y^2 = 1 \) at the angle \( \theta \), it is enough to conclude that, since \( x^2, y^2 \geq 0 \), for \((x, y)\) on the circle \( x^2 + y^2 = 1 \), \( y^2 \) must satisfy \( 0 \leq y^2 \leq 1 \), which means \(-1 \leq \sin(\theta) \leq 1 \).

(b) Graph \( y = \sin(x) \).
(c) What happens to the graph when I multiply \( \sin(x) \) by a constant real number \( A \)?

Algebraically, assuming \( A \geq 0 \), we have \(-A \leq A \sin(x) \leq A \). Graphing \( A \sin(x) \), we will see that the graph still “looks like” \( \sin(x) \) but stretched out in the \( y \)-directions.

(d) What does the graph \( \sin(3x) \) look like? (Hint: for \( f(x) = \sin(x), f(x + 2\pi) = f(x) \); what do we know about \( f(3x) \)?)

Multiplying the variable \( x \) first by three squishes the graph horizontally by a factor of three; we can see this by noting that \( \sin(3(x + \frac{2\pi}{3})) = \sin(x) \) so the period of this periodic function is cut by a third.

(e) For a real number \( \lambda \), what does the graph of \( \sin(x + \lambda) \) look like?

The entire graph is shifted horizontally by \( \lambda \).

(f) What does the graph of \( x^2 \sin(x) \) look like? (Hint: use part c above)

Well, we saw that, when we multiply by a constant \( A \), \( \sin(x) \) oscillates between \(-A \) and \( A \). When we multiply by the function \( x^2 \), \( \sin(x) \) oscillates between the function \( x^2 \sin(x) \).

(g) For a function \( f \) with domain all real numbers, what does \( f(x) \cdot \sin(x) \) look like?

Similar to above, \( f(x) \sin(x) \) oscillates between \( f(x) \) and \(-f(x) \). This makes seemingly complicated functions like \( e^x \sin(x) \) relatively easy to graph.