NAME __________________________

(1) Suppose $f(x) = 3\sin(x)$.
   (a) Give a domain on which $f(x)$ is one-to-one.

   We know that $\sin(x)$ is one-to-one on $[-\pi/2, \pi/2]$ and that $3^x$ is one-to-one on all real numbers, so we can use the domain $[-\pi/2, \pi/2]$ as a domain on which $f(x)$ is one-to-one.

   (b) Find $f^{-1}$ for your domain in part a (for partial credit, you may complete this step algebraically without finishing part a)

   We solve for $x$:
   
   \[
   y = 3^{\sin(x)} \\
   \log_3(y) = \sin(x) \\
   \arcsin(\log_3(y)) = x
   \]

   Therefore, $f^{-1}(x) = \log_3(\sin(x))$. The range of $f(x)$ on $[-\pi/2, \pi/2]$ is $[1/3, 3]$ (since the range of $\sin(x)$ on $[-\pi/2, \pi/2]$ is $[-1, 1]$ and the range of $3^x$ on $[-1, 1]$ is $[1/3, 3]$), so the domain of $f^{-1}(x)$ is $[1/3, 3]$.

   (c) What are the domain and range of your $f^{-1}$?

   The domain of $f^{-1}(x)$ is above. The range is given by the domain of the function $f(x)$ we inverted: $[-\pi/2, \pi/2]$.

(2) Reduce the following equation to a single real number (i.e., not involving logs) using the laws of logarithms:

   \[\log_5(15) + \ln(e) + \frac{1}{2} \log_5(81) + \log_2(8) - 3 \log_5(3)\]

Make as many simplifications as you can for partial credit.

\[\ln(e) = 1\] and, since $8 = 2^3$, $\log_2(8) = 3$. For the other three terms:

\[\log_5(15) + \frac{1}{2} \log_5(81) - 3 \log_5(3) = \log_5(15) + \log_5(81^{1/2}) - \log_5(3^3) = \log_5(15) + \log_5(9) - \log_5(27) = \log_5(15 \cdot 9) - \log_5(27) = \log_5 \left( \frac{15 \cdot 9}{27} \right) = \log_5(5) = 1\]

So

\[\log_5(15) + \ln(e) + \frac{1}{2} \log_5(81) + \log_2(8) - 3 \log_5(3) = 1 + 1 + 3 = 5\]
(3) Show that $\sin(\arccos(x)) = \sqrt{1-x^2}$.

Suppose we have a right triangle with side lengths $x$, 1, and $\sqrt{1-x^2}$ (so 1 is the hypotenuse). Then, for $\theta$ the angle across from the side with length $\sqrt{1-x^2}$, we have $\cos(\theta) = x/1 = x$ and $\sin(\theta) = \sqrt{1-x^2}/1 = \sqrt{x^2-1}$. So $\arccos(x) = \theta$ and therefore $\sin(\arccos(x)) = \sqrt{x^2-1}$. 