**Quiz #1 solutions**  
Math 220, section U1

**Instructions.** Be sure to show your work and explain your reasoning for full credit. No calculators are allowed.

**NAME ________________**

(1) Sketch the graph of a function with domain \((-\infty, 0) \cup [2, 4)\) and range \([1, 2] \cup (4, \infty)\)

We discussed how to do something like this in class. If you’re struggling with this, please let me know.

(2) Give the domain of the function:

\[
f(x) = \frac{\sqrt{x^2 - 3}}{\sqrt{8 - x^2}}
\]

The numerator has domain \((-\infty, -\sqrt{3}] \cup [\sqrt{3}, \infty)\) while the denominator has domain \([-\sqrt{8}, \sqrt{8}]\).

Since we must additionally avoid having a denominator of zero, only the open interval \((-\sqrt{8}, \sqrt{8})\) gives the allowable values to plug into the denominator. So, our overall domain is the set of things in both \((-\infty, -\sqrt{3}] \cup [\sqrt{3}, \infty)\) and \((-\sqrt{8}, \sqrt{8})\) which is \((-\sqrt{8}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{8}).\)

(3) Suppose you are painting a cube that was built to hold a ball of radius 10 ft. If the paint you are using covers 300 square feet per gallon, express the number of gallons of paint used to paint the cube as a function of the cube’s side length.

Let the variable \(s\) represent the side length of our cube. Then each side of the cube has area \(s^2\). There are 6 sides to the cube giving a total surface area of \(6s^2\) for the cube. Since the paint covers 300 square feet per gallon, we divide our surface area by 300 to get the total number of gallons used to cover the cube:

\[G(s) = \frac{6s^2}{300} = \frac{s^2}{50}.
\]

Finally, we need to determine what the domain of \(G(s)\) should be. Since the box was built to contain a ball of radius 10 ft, we must have that the side length of our cube is at least the diameter of the ball, or 20 feet. Therefore, the domain of \(G(s)\) should be \([20, \infty)\).

(4) Check whether the following functions are even, odd, or neither:

(a) \(f(x) = \frac{x}{|x|}\)

\[
f(-x) = \frac{-x}{|-x|} = -\frac{x}{|x|} = -f(x),
\]

so the function is odd.

(b) \(f(x) = x^3 - 4x^2 + 3x - 8\)

\[
f(-x) = (-x)^3 - 4(-x)^2 + 3(-x) - 8 = -x^3 + 4x^2 - 3x - 8 \neq \pm f(x).
\]

So \(f(x)\) is neither even nor odd.

Alternatively, note that \(f(1) = -8\) while \(f(-1) = -10\).

(c) \(f(x) = x^2 + x^4 + 5|x|\)

\[
f(-x) = (-x)^2 + (-x)^4 + 5|-x| = x^2 + x^4 + 5|x| = f(x),
\]

so \(f(x)\) is even.