Problem 1. Use a comparison to determine whether the integral converges or diverges.

\[ \int_{1}^{\infty} e^{-x^3} \, dx \]

**Solution.** On the interval \([1, \infty)\), we have that \(0 \leq e^{-x^3} \leq e^{-x}\). Now, \(\int_{1}^{\infty} e^{-x} \, dx\) converges (to \(e^{-1}\) in fact.) So by the Comparison Test, we have that the given integral will converge.

Problem 2. If \(\lim_{x \to \infty} f(x) = 1\) then \(\int_{0}^{\infty} f(x) \, dx\) diverges.

**Solution.** This is true. \(\lim_{x \to \infty} f(x) = 1\) means that for all \(\epsilon > 0\) there is some finite \(N > 0\) such that \(|F(x) - 1| < \epsilon\) whenever \(x > N\). Taking for instance \(\epsilon = \frac{1}{2}\), we have

\[ \int_{N}^{\infty} f(x) \, dx > \int_{N}^{\infty} \frac{1}{2} \, dx = \infty \]

So this integral must diverge.

Problem 3. If \(\lim_{x \to \infty} f(x) = 0\) then \(\int_{0}^{\infty} f(x) \, dx\) converges.

**Solution.** This is false! Consider \(f(x) = \frac{1}{x}\), \(\lim_{x \to \infty} f(x) = 0\), but the integral diverges by the \(p\)-test. (In fact both \(p\)-tests since \(\frac{1}{x}\) will not integrate nicely at infinity or at zero.)

Problem 4. \(\lim_{x \to 0} f(x) = \infty\) then \(\int_{0}^{1} f(x) \, dx\) diverges.
**Solution.** This is false! Consider \( f(x) = \frac{1}{x^{1/2}} \), \( \lim_{x \to 0} f(x) = \infty \) but in fact, 
\[
\int_0^1 f(x) \, dx = 2
\]
is convergent.

**Problem 5.** If \( f(-x) = -f(x) \) for all \( x \), then \( \int_{-\infty}^{\infty} f(x) \, dx = 0 \).

**Solution.** This is false! Recall that for such an integral you need to take two limits, so 
\[
\int_{-\infty}^{\infty} f(x) \, dx = \lim_{R \to \infty} \lim_{S \to -\infty} \int_S^R f(x) \, dx
\]
with \( f(x) = x \) for example, this limit will not exist. It does not exist since if \( R \) goes to infinity much faster than \( S \) goes to negative infinity, the integral should go to \( \infty \). Yet if \( S \) goes faster, the integral should go to \( -\infty \), and if they go at the same speed, the integral should go to \( 0 \). Thus, the limit will not exist and this integral diverges.