Ex: Prof. Pennings and his dog Elvis are at the beach. Elvis is 5 feet out in the water when a ball is thrown 100 ft away on the shoreline. What is the fastest Elvis can get the ball if he swims 4 ft/s and runs 10 ft/s?

\[ \text{Time} = \frac{\text{dist}}{\text{speed}} \]

\[ T(x) = \frac{100}{4} + \frac{100 - x}{10} \]

\[ \frac{dT}{dx}(x) = \frac{1}{4} \left( \frac{1}{2} \left( 25 + x^2 \right)^{-\frac{1}{2}} \cdot 2x \right) - \frac{1}{10} \]

\[ \frac{x}{4 \sqrt{25 + x^2}} - \frac{1}{10} = 0 \]

\[ \frac{x}{4 \sqrt{25 + x^2}} = \frac{1}{10} \]

\[ 10x^2 = \left( 4 \sqrt{25 + x^2} \right)^2 \]

\[ 100x^2 = 400 + 16x^2 \]

\[ 84x^2 = 400 \]

\[ x^2 = \frac{400}{84} \]

\[ x = \sqrt{\frac{100}{21}} \]

Here we are constrained to \( 0 \leq x \leq 100 \).

\[ T(0) = \frac{5}{4} + 10 = 11.25 \text{ s} \]

\[ T(100) = \frac{\sqrt{10000}}{4} \approx 25.03 \text{ s} \]

\[ T(\sqrt{\frac{100}{21}}) \approx 11.15 \text{ s} \]