Quiz VII
Sections 1.5, 1.6
October 26, 2007

NAME: ________________________________________________
(Please Print)

DIRECTIONS:

• Do each of the problems and show all work. No work means no points!
• Calculators ARE NOT ALLOWED on this quiz.
• Box or circle your and LABEL final solution.

SCORES:

1. _____________
2. _____________
3. _____________
**Problem 1.** Paulie’s Pizza Palace makes two different kinds of Pizza. Paulie’s Pepperoni Pizza uses 2 lbs cheese and 2 lb sauce while Paulie’s Cheese Pizzas uses 4 lbs cheese and 1 lb sauce. Paulie has 44 lbs of cheese and 26 lbs of sauce available. If Paulie sells each Pepperoni Pizza for $5 and each Cheese Pizza for $4, how many pizzas should he make to maximize his revenue?

**Solution.** We have the following information:

<table>
<thead>
<tr>
<th></th>
<th>Cheese</th>
<th>Sauce</th>
<th>Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pepperoni Cheese</td>
<td>2</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>Cheese</td>
<td>4</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Total</td>
<td>44</td>
<td>26</td>
<td>Maximize</td>
</tr>
</tbody>
</table>

Define the variables $x$ to be the number of pepperoni pizzas made and $y$ to be the number of cheese pizzas made. Our revenue function is $z = 5x + 4y$, and we wish to maximize this. We have the Pepperoni and Sauce Inequalities

$$2x + 4y \leq 44$$
$$2x + y \leq 26$$

As well as our “free” inequalities

$$x \geq 0$$
$$y \geq 0$$

When we intersect these lines, we get the points (0, 0), (0, 11), (22, 0), (0, 26) and (13, 0) and where the Pepperoni and Sauce boundary lines will intersect. Noting that (0, 0) satisfies all the inequalities, we will have that (0, 0), (0, 11), (13, 0) and where the Pepperoni and Sauce boundary lines will intersect will be our corner points.

To find where the last corner point is, we solve

$$2x + 4y = 44$$
$$2x + y = 26$$

Subtracting them, yields $3y = 18$, so $y = 6$. Plugging this into the second equation quickly yields $2x + 6 = 26$, so $x = 10$.

Now, we need only test our objective function on these points.

$$z(10, 6) = 5(10) + 4(6) = 74$$
$$z(0, 0) = 5(0) + 4(0) = 0$$
$$z(0, 11) = 5(0) + 4(11) = 44$$
$$z(13, 0) = 5(13) + 4(0) = 65$$

So Paulie wants to make 10 Pepperoni and 6 Cheese Pizzas.
Problem 2. Find where the function \( z = 15x - 9y \) is maximized on the pentagonal domain with vertices \((-1, 1), (4, 0), (1, -3), (0, -5) \text{ and } (1, 1)\).

Solution. Test the function at these endpoints.

\[
\begin{align*}
z(-1, 1) &= 15(-1) - 9(1) = -24 \\
z(4, 0) &= 15(4) - 9(0) = 60 \\
z(1, 3) &= 15(1) - 9(3) = -12 \\
z(0, -5) &= 15(0) - 9(-5) = 45 \\
z(1, 1) &= 15(1) - 9(1) = 6
\end{align*}
\]

So the maximum occurs at \((4, 0)\).
Problem 3. Indicate where \( z = -2x + y \) is minimized subject to the constraints

\[
\begin{align*}
    x &\geq 5 \\
y &\geq 7 \\
x &\leq 7 \\
y &\leq 9
\end{align*}
\]

Solution. We are working on a square with corner points (5, 7), (7, 7), (7, 9) and (5, 9). We need only test our objective function on these points.

\[
\begin{align*}
z(5, 7) &= -2(5) + 7 = -3 \\
z(7, 7) &= -2(7) + 7 = -7 \\
z(7, 9) &= -2(7) + 9 = -5 \\
z(5, 9) &= -2(5) + 9 = -1
\end{align*}
\]

So our minimum occurs at (7, 7) and is \(-7\). \(\square\)