Quiz V

Sections 7.1, 7.2

October 5, 2007

NAME: ____________________________________________
(Please Print)

DIRECTIONS:

• Do each of the problems and show all work. No work means no points!
• Calculators ARE NOT ALLOWED on this quiz.
• Box or circle your and LABEL final solution.
• Cheer the Cubbies on in the playoffs!

SCORES:

1. __________
2. __________
3. __________
Problem 1. Give a decimal answer using the provided table:
For a Bernoulli Process with $n = 12$ and $p = .35$, find the probability of getting
a) at most 5 successes.
b) at least 6 successes.
c) 4, 5, or 6 successes.

Solution. a) This is done simply by reading from the table. We first find the
columns in the table corresponding to $n = 12$, here $x = 5$ and $p = .35$. Reading
this number off the table, we arrive at $B(5) = .7873$.

b) The table tells us at most $x$ successes, so we must consider the complement of
at least 6 successes. If we don’t have at least 6 successes, we have at most 5 suc-
cesses (this we can find on the table). From part a), we have that $B(5) = .7873$, so
Pr(at least 6 successes) = $1 - B(5) = 1 - .7873 = .2127$.

c) We want to find $b(4) + b(5) + b(6)$. Recall that:
$B(6) = b(0) + b(1) + b(2) + b(3) + b(4) + b(5) + b(6)$ and
$B(3) = b(0) + b(1) + b(2) + b(3)$, so we have
$b(4) + b(5) + b(6) = B(6) - B(3)$.
We can find both of these numbers on the table provided, $B(6) = .9154$ and
$B(3) = .3567$, so
$b(4) + b(5) + b(6) = .9154 - .3567 = .5587$. □
Problem 2. Two coins are taken at random without replacement from a box containing 3 pennies and 2 nickels.

a) Define an appropriate random variable \( X \) for this problem.
b) Find the probability distribution for \( X \).
c) Find the expected value of \( X \).

Solution. a) An appropriate random variable would be the value of the two coins.

b) Our random variable \( X \) can take on the values 2, 6 and 10.

\[
\begin{array}{|c|c|c|}
\hline
X = x_i & \text{Outcomes} & \text{Probability} \\
\hline
2 & (p,p) & \frac{3}{5} \cdot \frac{2}{4} \\
6 & (p,n), (n,p) & \frac{3}{5} \cdot \frac{2}{4} + \frac{2}{5} \cdot \frac{3}{4} \\
10 & (n,n) & \frac{2}{5} \cdot \frac{1}{4} \\
\hline
\end{array}
\]

c) The expected value \( E(X) = \sum x_i p_i \), so
\[
E(X) = 2\left(\frac{3}{5} \cdot \frac{2}{4}\right) + 6\left(\frac{3}{5} \cdot \frac{2}{4} + \frac{2}{5} \cdot \frac{3}{4}\right) + 10\left(\frac{2}{5} \cdot \frac{1}{4}\right).
\]

\( \square \)
Problem 3. An urn contains 15 Orange balls and 25 blue balls. Find the expected number of Orange balls if 15 balls are drawn
a) with replacement.
b) without replacement.
Clearly indicate what formula or theorem you are using.

Solution. a) with replacement guarantees that our probabilities are the same for each trial, that is time we draw a ball from the urn. Thus we have a Bernoulli Process with $n = 15$ trials and $p = \frac{15}{40}$, so we expect
\[ \mu = np = (15)\left(\frac{15}{40}\right) \] Orange Balls.

b) without replacement is covered by a theorem from class, we have $\mu = n\left(\frac{a}{a+b}\right)$ where $n$ is the number of balls drawn, $a$ is the number of Orange and $b$ is the number of Blue. So,
\[ \mu = n\left(\frac{a}{a+b}\right) = 15\left(\frac{15}{40}\right). \]