Quiz IV
Sections 6.4, 6.5
September 28, 2007

NAME: ____________________________________________
(Please Print)

DIRECTIONS:

• Do each of the problems and show all work. No work means no points!
• Calculators ARE NOT ALLOWED on this quiz.
• Box or circle your and LABEL final solution.

SCORES:

1. __________
2. __________
3. __________
Problem 1. Given $\Pr(E \cup F) = .6$ and $\Pr(E) = .25$, find $\Pr(F)$ so that $E$ and $F$ will be independent.

Solution. We need to recall the inclusion exclusion formula and what it means to be independent. Requiring independence gives us that

$$\Pr(E \cap F) = \Pr(E) \Pr(F) = .25 \Pr(F).$$

Plugging this into the inclusion exclusion formula yields $\Pr(E \cup F) = \Pr(E) + \Pr(F) = \Pr(E \cap F)$, so

$$.6 = .25 + \Pr(F) - .25 \Pr(F).$$

So, $.75 \Pr(F) = .35$, so $\Pr(F) = \frac{.35}{.75} = \frac{45}{75}$. \hfill \Box
Problem 2. An experiment is performed 12 times with probability of success .35. Find the probability of getting exactly 5 successes.

Solution. This is a Bernoulli process if we define getting Tails to be a success. Here we have \( p = .35 \), \( q = 1 - .35 = .55 \) so Bernoulli’s Formula tells us that 
\[
\Pr(k \text{ successes}) = C(n, k)p^kq^{n-k}.
\]
So,
\[
\Pr(5 \text{ successes}) = C(12, 5)(.35)^5(.55)^{12-5} = C(12, 5)(.35)^5(.55)^7.
\]
Problem 3. The Chicago Bears have three quarterbacks. Grossman plays 60% of the time, Griese 30% and Orton 10%. The probability that they fumble on a given play is .07, .02 and .05 respectively. Find the probability that a fumble is due to Grossman.

Solution. Let $F_1$ be Grossman, $F_2$ be Griese and $F_3$ be Orton. Let $E$ denote the event that the ball is fumbled. Noting that:

$$\Pr(E) = \Pr(F_1)\Pr(E|F_1) + \Pr(F_2)\Pr(E|F_2) + \Pr(F_3)\Pr(E|F_3)$$

$$= (.6)(.07) + (.3)(.02) + (.1)(.05) = .042 + .006 + .005 = .053$$

Using Bayes’ Theorem, we have:

$$\Pr(F_1|E) = \frac{\Pr(F_1)\Pr(E|F_1)}{\Pr(E)}$$

$$= \frac{(.6)(.07)}{.053}$$

$$= \frac{.042}{.053}$$

$$\approx .792$$

If the quarterback fumbles, the probability that it was Grossman is about .792.