Permutations vs. Combinations

It is **very** important to make the distinction between permutations and combinations. In permutations, order matters and in combinations order does **not** matter. The important information can be summarized by:

	Order	Number
Permutation	matters	$P(n,k) = \frac{n!}{(n-k)!}$
Combination	does not matter	$C(n,k) = \frac{n!}{(n-k)!k!}$

Examples:

A company has to select 3 officers from a pool of 6 candidates. How many different ways can this be done if:
(a) The officers are distinct?
(b) The officers are not distinct?

It is **very** important whether or not these officers are distinct.
(a) If the officers are distinct, we are picking a triple (s_1, s_2, s_3) with each s_i being a candidate, and order matters. This means we are finding a 3-permutation from a set of 6 elements. So there are: $\mathrm{P}(6,3) = \frac{6!}{(6-3)!} = \frac{6!}{3!} = 6 \cdot 5 \cdot 4 = 120$ distinct ways to pick these officers.

(b) If the officers are not distinct, the triples (s_1, s_2, s_3), (s_1, s_3, s_2), (s_3, s_2, s_1), etc. are the same since the positions are the same. So, we are finding a 3-combinations from a set of 6 elements. So there are:
$\mathrm{C}(6,3) = \frac{6!}{(6-3)!3!} = \frac{6!}{3!3!} = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = 5 \cdot 4 = 20.$