Chapter 5

Section 5.1

Algebra of Sets: Know the definitions that are applicable to sets. How to use set-builder notation to define a set. The difference between being an element of a set and a subset of a set. The three main set operations: union, intersection and complement. Using Venn Diagrams to illustrate sets and set operations. Counting sets, using the inclusion-exclusion formula for 2 sets, 3 sets and the pattern for more sets.

Section 5.2

Multiplication Principle and Permutations. Know the definitions for this section. Breaking up complicated (multi-step) experiments into a sequence of steps and using the multiplication principle to figure out the total number of outcomes. Using a tree diagram to illustrate an experiment. Samples and ordered samples. Sampling \( k \) elements from a set of \( n \) elements: with replacement \( (n^k) \) and without replacement \( (n(n−1)\cdots(n−k+1)) \). Permutations and factorials. Calculating \( P(n, k) \), the number of \( k \)-permutations. Using permutations and the multiplication principle to find the number of arrangements that can be made of distinct types of objects.

Section 5.3

Combinations. Know the definitions for this section. Noting that in permutations order matters and in combinations order does not matter. Relation of unordered samples and subsets. \( k \)-subsets of a set of \( n \) elements. Calculating \( C(n, k) \) and how \( C(n, k) \) and \( P(n, k) \) relate to each other. \( C(n, k) = C(n, n−k) \) and why. Words.
Chapter 6

Section 6.1
Probability Spaces. Know the definitions for this section. Sample spaces, how to draw pictures to illustrate them and how sample spaces and events relate ($E \subset S$). Properties of probability functions. Interpreting probability functions via relative frequency interpretation. Determining probability functions deductively versus determining them empirically. How to calculate the probability of an event. The idea of “equal likelihood” and “at random.”

Section 6.2
Calculation of Probabilities. Know the definitions for this section. How probability and set operations interact with each other. Mutual exclusivity. The complementation property ($Pr(E') = 1 - Pr(E)$.) The birthday problem, and why the odds of two people in a group having the same birthday “seems to be too high.” The difference between a relative complement and the normal definition of complement. The inclusion-exclusion property for probabilities. Using Venn Diagrams to calculate probabilities.

Section 6.3
Conditional Probability. Know the definitions from this section. How conditional probability differs from regular probability, both in terms of the formula and what differs in the definitions. Conditional Probability calculations ($Pr(E|F) = \frac{n(E\cap F)}{n(F)}$) The multiplication rule for conditional probability. The use of tree diagrams, namely probability trees to illustrate multi-step experiments and calculate probabilities. The idea and calculation of “total probability.”

Section 6.4
Bayes' Theorem. How to calculate the probability of the result of a step in a multistep experiment given a specific result. Note that, in general, $Pr(E|F) \neq Pr(F|E)$.

Bayes’ Theorem: $Pr(F_j|E) = \frac{Pr(F_j)Pr(E|F_j)}{Pr(E)}$. The Transfer Problem.

Chapter 6

Section 6.5
Independence and Repeated Trials: Know the definitions for this section. Two events $E$ and $F$ are independent if $Pr(E \cap F) = Pr(E)Pr(F)$, alternatively $Pr(E|F) = Pr(E)$. There is a theorem about independence of events and their complements. Independence of $n$ events when $n > 2$. Trials and repeated trials, specifically applications with Bernoulli Trials. How to reduce any experiment
to a Bernoulli Trial. Bernoulli Processes and Bernoulli’s Formula
Pr(k successes) = C(n, k)p^k q^{n-k}.

Chapter 7

Section 7.1
Random Variables and Probability Distributions. Know the definitions for this section. Discrete, continuous and infinite random variables and the differences between them. How a probability distribution can be defined for a random variable. Bernoulli random variables and the binomial distribution. $b(k) = C(n, k)p^k q^{n-k}$. Drawing a histogram for a given distribution. The cumulative distribution function $B(x)$, often given as a table. The relation between the binomial distribution and the cumulative distribution.

Section 7.2
Expected Value. Know the definitions for this section. For a random variable $X$ taking on values $x_i$ with probability $p_i$, $E(X) = x_1p_1 + x_2p_2 + \cdots + x_np_n$. The concept of a fair game, the expected value is zero. Means of distributions. For a binomial distribution of $n$ trials with probability of success $p$, the mean is given by $\mu = np$. Sample means, $\bar{x} = \frac{x_1 + \cdots + x_n}{n}$. For an urn containing $a$ red balls and $b$ blue balls, the expected number of red balls when you draw $n$ balls from the urn without replacement is $\mu = \frac{n}{a+b}$.

Chapter 1

Section 1.2
Lines in the plane. Know the definitions for this section. Slope and intercepts of a line. Finding the slope given two points. Using the slope-intercept form to find the equation of a line given a point and the slope. The slope-intercept form of a line, most often how a line is given when graphing. ($y = mx + b$ form).

General Linear Form of a line. How to go from one form of a line to another. Two lines are parallel if they have the same slope (and are distinct). Two lines are perpendicular if their slopes multiply out to $-1$.

Section 1.3
Systems of Equations in Two Variables. Know the definitions for this section. Systems of equations have either 0, 1 or infinitely many solutions, geometrically, this is because two lines can cross at some point, are parallel, or are the same line. Consistent versus inconsistent systems and what type of lines they correspond
to. Using the method of elimination to solve systems of equations. Breakeven analysis. Fixed, unit and total costs.

Section 1.4
Linear Inequalities and Half-Planes. Know the definitions for this section. The difference between open ($<$ or $>$) and closed ($\leq$ or $\geq$) half-planes. Closed half-planes contain their boundary line. Finding corner points and using test points to graph a feasibility region. Polygonal wedges versus polygons in feasibility regions.

Section 1.5
Linear Programming in the Plane. Know the definitions for this section. A linear function in $x$ and $y$ is $z = ax + by$. Lines of constancy and lines of constant revenue. For a linear function and a polygon $P$ in the plane, maximum and minimum values of the linear function on $P$ occur at the corner points. Linear Programming to optimize a linear function subject to constraints by first determining the feasibility region and then checking the values of the function on the corner points.

Chapter 1
Section 1.6
Applications of Linear Programming: Know the definitions for this section. Three new types of problems, “Production Schedules,” “Maximizing Revenue,” “Diet Problem.” These problems can all be dealt with the same way, by defining variables, and then an objective function. We then determine if we want to maximize or minimize the objective function and find our constraints (look for “at most” or “at least). Then finding corner points on which we test our objective function.

Chapter 2
Section 2.1
Row Operations and Gaussian Elimination: Know the definitions for this section. There are precisely three Elementary Row Operations: switching rows, multiplying a row by a non-zero constant and adding a multiple of another row to a given row. Representing a system of linear equations by an augmented matrix. The difference between a row and a column. Row Echelon Form. Using Gaussian Elimination to get a matrix into Row Echelon Form.
Section 2.2

More on Gaussian Elimination: Know the definitions for this section. Back Substitution to get solutions to a system from a matrix in REF. Reduced Row Echelon Form (the best form). The Reduced Row Echelon Form of a matrix is unique. Multisystems, two systems of equations with the same coefficients on the variables, but different constant terms. Using matrices to solve a multisystem.

Section 2.3

Consistency of Systems: Know the definitions for this section. Consistent and Inconsistent systems. A row like 0 0 . . . 0|c with c \neq 0 being the defining characteristic of an inconsistent system in REF. Inconsistent if the corresponding system has zero solutions, consistent if the corresponding system has one or infinitely many solutions. Row Rank of a matrix, needing to be in REF to determine the row rank of a matrix. A linear system is consistent if and only if the row rank of the augmented matrix is equal to the row rank of the augmented matrix. The use of parameters for free variables when a system has infinitely many solutions. A row of zeroes or more variables than equations say that we will have to use parameters if the system is consistent.

Section 2.4

Gauss-Jordan Elimination: Know the definitions for this section. Pivoting, that is picking a non-zero entry and row operation on your matrix to make this entry, the pivot, a 1 and everything else in its column a zero. Gauss-Jordan Elimination is simply using the pivot repeatedly to get a matrix in RREF. (If we have n equations/rows, we will have to perform the pivot n times to get into RREF.)

Chapter 3

Section 3.1

Matrix Operations. Know the definitions for this section. The dimensions of a matrix tell us the number of rows and columns. An $m \times n$ matrix has $m$ rows and $n$ columns. Two matrices are equal if they have the same dimension and all their corresponding entries are equal. The transpose of a matrix. Matrix addition (it works as nicely as we could hope for). Multiplication of a matrix by a scalar and matrix multiplication. Being careful with the dimensions of matrices as there are matrices A and B so that $AB$ is a matrix, but $BA$ does not make sense. Multiplication of matrices is NOT commutative, $AB \neq BA$ in general. Square matrices and taking powers of a square matrix.
Section 3.2

Inverse of a Matrix. Know the definitions for this section. The Cancellation Property, $AB = AC$ implies $B = C$, only holds if $A$ is invertible. The identity matrix and uniqueness of inverses. Inverses of products, $(AB)^{-1} = B^{-1}A^{-1}$.

How to calculate the inverse of any square matrix via row operations. Using the inverse of the coefficient matrix to solve linear systems and uniqueness of this solution. The formula for the inverse of a $2 \times 2$ matrix.

Section 3.3

Leontief Open Model. Know the definitions for this section. How to determine the consumption matrix, $C$, based on information from a word problem. The demand vector $D$ and production vector $X$. Solving for the production vector with $X = (I - C)^{-1}D$. The concept of productive and profitable economies.

For more information, please go to: