INTRO GRAPH THEORY, FALL 2017 - HOMEWORK 1

WARMUP PROBLEMS: Section 5.1: #1–4, 6–8, 11. Section 5.2: #1, 2, 3, 5, 6, 7. Do not write up! Think about how to solve them to make sure you understand the material.

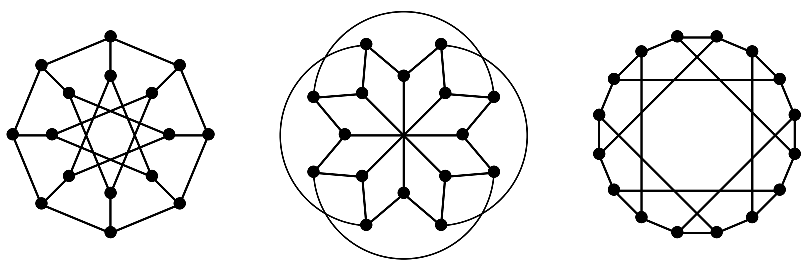
EXTRA PROBLEMS: Section 5.1: #13, 14, 17, 20, 23, 25, 27, 34, 38. Section 5.2: #8–10, 13, 14, 17, 19. Do not write up! If you have time, think about some of these.

WRITTEN PROBLEMS: Solve five of the six problems below. Written solutions (in English) due Thursday, Sep. 21, in class. First problem session Tuesday, Sep. 19, in class. Try to solve the problems BEFORE then. Second lecture SUNDAY, Sep. 17, 6:40PM.

Warmup or extra problems may appear on the examination. Written problems require proofs of why the claim or answer is correct. Solutions must be in English, in sentences. Students must write homework solutions by themselves. A request to do something for n-vertex graphs means for all n. Constructing an example includes giving proof. Results from class may be used if stated correctly; you must say what result you use.

The text has many other problems to help you practice and learn. Those marked ($-$) are easier and check understanding. Those marked ($\diamondsuit$) are instructive or important.

1. Determine which pairs of graphs below are isomorphic.



2. Prove that the Petersen graph has no cycle of length 7.

3. A *subcube* of dimension r in Q_k is a subgraph induced by 2^r vertices that agree on some $k-r$ coordinates.

(a) Prove that every cycle of length $2r$ in Q_k lies in a subcube of dimension at most r. Prove that this subcube is unique when $r \in \{2,3\}$ but need not be unique when $r = 4$.

(b) Count the copies of C_4 and C_6 in Q_k. Also count the copies of C_8 contained in 3-dimensional subcubes.

4. Prove that every k-regular graph with girth 6 has at least $2k^2-2k+2$ vertices. (Comment: The graph in Exercise 5.1.36 is an example achieving equality when $k = 3$.)

5. Let G be a graph with average vertex degree a.

(a) In terms of $d(x)$, determine when $G - x$ has average degree at least a. Conclude that if $a > 0$, then G has a subgraph with minimum degree greater than $a/2$.

(b) For $\varepsilon > 0$, show that a subgraph with minimum degree greater than $(a/2) + \varepsilon$ cannot be guaranteed. For example, when $\varepsilon = 1/3$, a graph having no subgraph with minimum degree greater than $a/2 + \varepsilon$ is P_3.

6. Let p and q be nonnegative integer lists, with $p = (p_1, \ldots, p_r)$ and $q = (q_1, \ldots, q_s)$. The pair (p, q) is *bigraphic* if some bipartite graph has $p_1, \ldots, p_r$ as the vertex degrees in one part and $q_1, \ldots, q_s$ as the degrees in the other. When $\sum p_i > 0$, prove that (p, q) is bigraphic if and only if (p', q') is bigraphic, where (p', q') is obtained from (p, q) by deleting the largest element k from p and subtracting 1 from the k largest elements of q.