Classic Problem. Proposed by the editors. A lion and a man are in an enclosure. The maximum speed of the lion is equal to the maximum speed of the man. Can the lion catch the man?

Solution. If the lion and the man start at the same location, then of course the lion catches the man immediately. We assume now that they start at different locations and show that the man can evade capture forever.

If the man starts on the boundary of the enclosure, he first moves into the interior. As long as he does this by traveling less than half the distance to the lion, he won’t be caught during this step. Once he is in the interior, we can let \( D \) be an open disk centered at the man’s location that is entirely contained in the enclosure. We now give a strategy that the man can follow to evade capture while staying inside \( D \), and therefore inside the enclosure.

Let the unit of distance be chosen so that \( D \) has radius 2, and let the unit of time be chosen so that the maximum speed of both lion and man is 1. The strategy proceeds in stages. In stage 1, the man starts running directly away from the lion and runs at maximum speed in a straight line for 1 unit of time. Since the lion cannot run faster than the man, the man cannot be caught during stage 1. For \( n \geq 2 \), at stage \( n \) the man travels at maximum speed a distance \( 1/n \) in a direction that is perpendicular to the line \( L \) that passes through his location at the beginning of the stage and the center of \( D \). There are two such directions to choose from, and the man chooses based on the location of the lion. If the lion is in one of the half planes determined by \( L \), then the man runs into the other half plane. The man can run either way if the lion is on \( L \). Every point that the man visits during stage \( n \) is closer to the man’s position at the beginning of the stage than it is to the lion’s position, so the man evades capture during stage \( n \).

The time elapsed during the first \( n \) stages is \( \sum_{k=1}^{n} 1/k \), which diverges. But the distance between the man and the center of \( D \) after \( n \) stages, by repeated use of the Pythagorean theorem, is \( \sqrt{\sum_{k=1}^{n} 1/k^2} \), which converges and in particular is bounded (generously) by 2. Thus the man evades capture forever while remaining inside \( D \).

Editorial comment. We have treated the lion and man as points and assumed that to capture the man, the lion must reduce the distance between them to zero. The solution given shows that certain details of the problem don’t matter, such as the shape of the enclosure or the initial positions of the man and lion (as long as they are distinct).

The problem has a colorful history. It was proposed by Richard Rado in the 1930s, with the enclosure being a disk, and solved by Abram Besicovitch in 1952. The problem was popularized by John Littlewood in his book A Mathematician’s Miscellany (see B. Bollobás, ed. (1986), Littlewood’s Miscellany, Cambridge: Cambridge Univ. Press, pp. 114–117). For further details and generalizations see Bollobás, B., Leader, I., and Walters, M. (2012), Lion and man—can both win?, Israel J. Math. 189: 267–286.

It is tempting to think that the man’s best strategy is to stay as far from the lion as possible, and in the case of a circular enclosure this means that the man would run to the boundary and then run around the boundary (perhaps sometimes changing direction). However, if the man stays on the boundary then the lion can catch the man by running outward from the center of the enclosure while staying on the radius from the center to the man. Thus, in order to avoid capture, the man must step into the interior of the enclosure. This gives him the freedom to move in
any direction—a freedom that is exploited in Besicovitch’s solution.

[Possible additional topic for editorial comment: In the solution above, the man’s velocity is discontinuous, but a small modification makes his velocity continuous. In the modified solution, the man begins and ends each stage at rest. At the beginning of stage \( n \) he accelerates to maximum speed in time \( \epsilon_n \), and at the end he decelerates to a stop in time \( \epsilon_n \). If \( \epsilon_n \) is sufficiently small, the man still evades capture.]