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GOSSIPING WITHOUT DUPLICATE TRANSMISSIONS*

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Abstract. n people have distinct bits of information, which they communicate via telephone calls in which they transmit everything they know. We require that no one ever hear the same piece of information twice. In the case 4 divides n, $n \geqq 8$, we provide a construction that transmits all information using only $9n/4-6$ calls. Previous constructions used $\frac{1}{2}n \log n$ calls.

The original gossip problem asks for the minimum number of calls permitting a complete passage of information from each person to every other in some group. The answer of $2n-4$ for $n \geqq 4$ has been demonstrated in numerous ways, e.g., [1], and the optimal solutions have been characterized [2], [3]. In [5], we added an additional requirement, that no one hear his own original piece of information in the course of the calling scheme. This is impossible to achieve if n if odd, but if n is even, $2n-4$ calls still suffice, and [5] characterized these solutions.

Next we can prohibit anyone hearing any given piece of information more than once. This implies that no-one hears his own information. If $n \equiv 2 \bmod 4$, then whether it is ever possible to transmit all information under this restriction remains an open question. ($n = 6$ or 10 can be shown impossible without much difficulty.) For 4 divides n, H. W. Lenstra et al. [4] provided an inductive construction that succeeds. If $n/4 \equiv -k \bmod 4$, they divide the people into three groups of size $n/4+k$, $n/4+k$ and $n/2-2k$, each divisible by 4. Forming $n/4$ mini-groups of four people with two from one group and one from the other two, they perform three calls on each. This is done so that in each of the three large groups, all n pieces of information are known by exactly one person. Then they perform induction. If $f(n)$ is the number of calls used, this gives $f(n)=3n/4+2f(n/4+k)+f(n/2-2k)$. This is satisfied by $f(n) \approx \frac{1}{2}n \log n$. (That is exactly the solution if n is a power of 2.)

In this note, we provide an explicit construction for $n \geqq 8$, using only $9n/4-6$ calls. It would be nice to show this is optimal. The best current lower bound is $2n-3$ for $n>8$, as remarked in [6].

The construction. We begin by dividing the people into $n/4$ groups of 4. In each group, we perform four calls in a square so that each knows all four tidbits from his group. Label the points x_{ij} for $1 \leqq i \leqq n/4$, $1 \leqq j \leqq 4$.

Arrange the squares around a circle, with two points on the inner ring and two on the outer, as in Fig. 1a. We will leave the outer points as they are, knowing 4 pieces of information, until the end. The points on the inner ring will accumulate $n-4$ pieces in such a way that they can then be matched to the outer points.

Label the points in the ith square x_{i1}, x_{i2}, x_{i3}, x_{i4}, so that x_{i1} and x_{i2} are on the inner circle. $x_{1,1}$ and $x_{n/4,1}$ will be special points. We perform in order the calls $(x_{1,2}, x_{2,1})$, $(x_{1,2}, x_{3,1})$, $\cdots$, $(x_{1,2}, x_{n/4-1,1})$ and, also in order, the calls $(x_{n/4,1}, x_{n/4-1,2})$, $(x_{n/4,1}, x_{n/4-2,2})$, $\cdots$, $(x_{n/4,1}, x_{2,2})$. (See Fig. 1b.) In each sequence, four additional bits of information are involved on each call. For $1<k<n/4$, afterwards $x_{k,1}$ knows all information in $\{x_{ij}: i \leqq k, 1 \leqq j \leqq 4\}$ and $x_{k,2}$ knows all in $\{x_{ij}: i \geqq k, 1 \leqq j \leqq 4\}$, $x_{1,1}$ and $x_{n/4,2}$ still know the four bits they began with, while $x_{1,2}$ knows everything except

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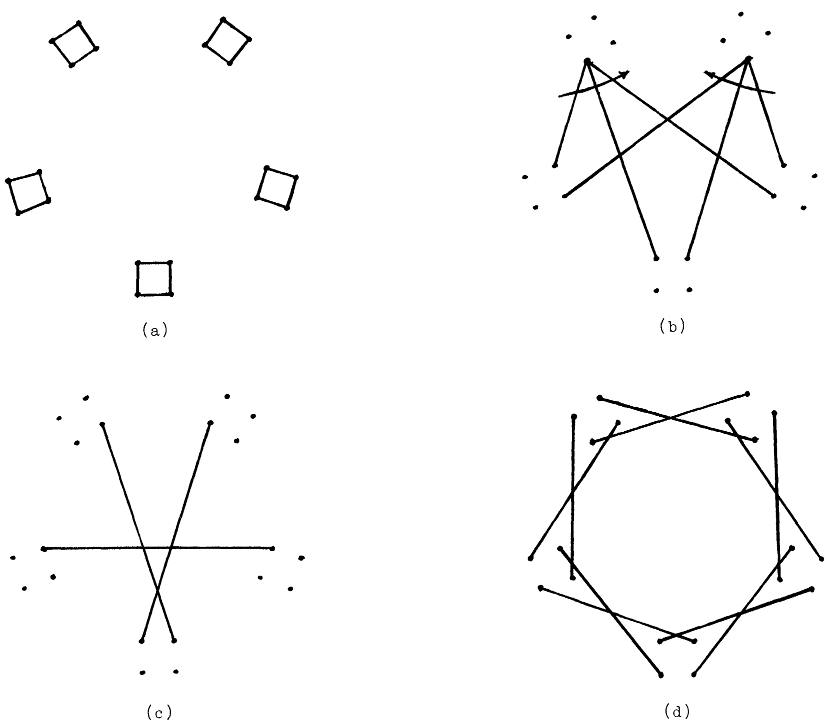

FIG. 1

$\{x_{n/4,j}\}$, and $x_{n/4,1}$ knows everything except $\{x_{1j}\}$. Note that four points $x_{1,2}$, $x_{n/4-1,1}$, $x_{n/4,1}$ and $x_{2,2}$ already know $n-4$ pieces of information.

In the third phase, $x_{k-1,1}$ and $x_{k+1,2}$ call each other, for $2 \leqq k \leqq n/4-1$. (See Fig. 1c.) The former knows the "lowest" $4(k-1)$ pieces of information and the latter the "highest" $4(n/4-k)$ pieces. Together they now know all but $\{x_{kj}: 1 \leqq j \leqq 4\}$.

Finally, the two inside points, knowing all but $\{x_{kj}\}$, are matched with the two outside points, knowing only $\{x_{kj}\}$, for $1 \leqq k \leqq n/4$. This completes the construction.

It is easy to see that no pair of points both knowing any given piece of information ever speak to each other, so there are no duplicate transmissions, and at the end everyone knows everything. Summing up the number of calls used in each of the four stages, we have $n+2(n/4-2)+(n/4-2)+n/2=9n/4-6$ total calls.

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