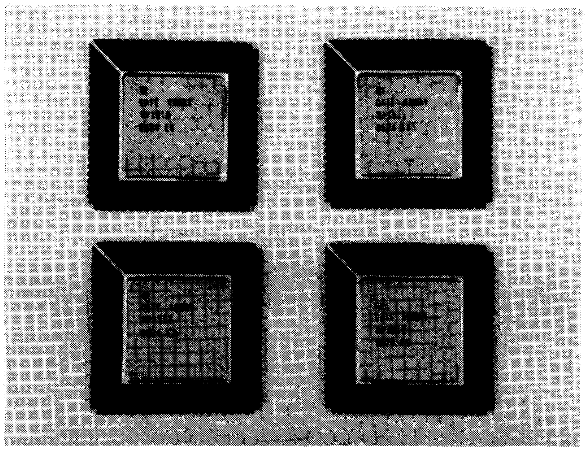

Fig. 7. Four 3 μm CMOS chips fabricated by GE to verify the 2-D FFT design. These include the memory crossbar switch, adder/subtractor, and partitioned multiplier chips.

Butterfly Processor

There are 16 butterfly processors in the one-dimensional FFT processor. The butterfly processors (see Fig. 6) can compute a radix two decimation in frequency butterfly on two complex data points, as well as performing an ordinary data transfer from one of its ports to the other as needed in the "dummy pass."

The left and right ports are bidirectional 32-bit wide data paths. The buffers and multiplexers enable the left and right ports to be used as inputs or outputs. These ports connect to the working memories. When data flow from the left working memory to the right one, the left port is configured as an input while the right port is an output. These data ports are time multiplexed to allow both operands to be received through the same port while both results are transmitted through the other port. The twiddle factors for the butterfly processor are input through a 16 bit time-multiplexed port connected to the coefficient memory.

The butterfly unit implements the butterfly equations using 16-bit two's complement integer arithmetic. This unit consists of four two's complement multipliers, three adders, and three subtractors. The output of the multiplier is truncated to the most significant 16 bits. This moves the binary point one position to the right after each butterfly operation in order to prevent a data overflow. The image is later scaled by a barrel shifter in the image memories to preserve the maximum possible resolution.

Conclusion

An architecture has been developed which can perform two-dimensional fast Fourier transforms at video rate. This architecture utilizes both pipelining and parallel processing to increase performance. The architecture can meet a wide range of performance by simply changing the total number of butterfly processors. With 16 butterfly processors, the expected performance is comparable to 63 million 16 bit fixed point multiplications per second. Using the two-dimensional FFT processor in a pipeline of image processors, video rate image transformations are possible.

The heart of the processor, a fast, pipelined one-dimensional FFT processor, has been designed using gate arrays. These gate arrays have been fabricated and tested. (See Fig. 7.)

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On the Construction of Communication Networks Satisfying Bounded Fan-In of Service Ports

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***Abstract*—This correspondence addresses the problem of minimizing the number of service ports of a central facility which serves a number of users subject to some constraints. At any time, a set of at most s users may want to use the facility, and one user can be connected to each port at a given time. We assume there are direct communication links from users to service ports, with at most d links incident at a single service port. This problem maps to the graph-theoretic problem of minimizing the number of outputs of a bipartite graph with n inputs, such that the degree of each output node is at most d and every set of $k \leqslant s$ inputs is joined collectively to at least k different outputs. We denote this minimum by $f(n, s, d)$. We define $\theta(s, d) = \lim_{n \to \infty} f(n, s, d)/n$. A general construction yields $\theta(s, d) \leqslant s/(s + d - 1)$, and this is shown to be optimal when $s \leqslant 2$ or $d \leqslant 2$. For larger values of s and d, better constructions are given. When $s \leqslant d^r$, then we show that $\theta(s, d) \leqslant r \lceil s^{1/r}/r \rceil / d$. A linear programming formulation is also given.**

***Index Terms*—Bipartite, bounded concentrator, communication networks, degree-restricted, expander graph, matching, service facility.**

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I. Introduction

Imagine n users trying to communicate with a central facility that has m service ports. At any time, a set of at most s users may want to use the facility, and one user can be connected to each port at a given time. We assume there are direct communication links from users to service ports, with at most d links incident to a single service port. This constraint on degree may model a fan-in limitation of some circuitry at the service port. The problem is to satisfy the constraints with the minimum number of service ports.

More formally, given positive integers n, s, and d, we seek the minimum m allowing a bipartite graph with n input nodes, m output nodes of degree at most d, and each set of s inputs matchable to s distinct outputs. By Hall's Theorem on matching in bipartite graphs, this is the same as having each set of $k \leqslant s$ users joined to at least k outputs. Call this the (s, d)-matching problem, and let $f(n, s, d)$ be the minimum value of m.

The efficiency of the construction is described by the ratio m/n, so we let $\theta(n, s, d) = f(n, s, d)/n$. It also makes sense to define $\theta(s, d) = \lim_{n\to\infty} \theta(n, s, d)$, because a solution for any given (n_0, s, d) can be replicated, with the leftover inputs connected straight across, to obtain $\theta(n, s, d) < \theta(n_0, s, d) + n_0/n$. This is useful in proving lower bounds, because for $\theta(s, d)$ we may assume the components of an optimal graph all have the same input–output ratio.

In Section III, we present a simple construction for general (s, d) yielding $\theta(s, d) \leqslant s/(s + d - 1)$, which is of interest only because it is optimal for $s \leqslant 2$ and $d \leqslant 2$. Here we also present a construction useful whenever s is not arbitrarily larger than d, i.e., $s \leqslant d^r$. This yields $\theta(d^r, s, d) \leqslant r\lceil s^{1/r}/r\rceil/d$. Compare this to the trivial lower bound $\theta(s, d) \geqslant 1/d$. If s is bounded in terms of d by $\log s \leqslant \log d \log\log d$, then putting $r = \lceil \log d\rceil$ yields $\theta(d^r, s, d) \leqslant \lceil \log d\rceil/d$. We have not chosen the base of the logarithm here; choosing the largest base that satisfies $\log s \leqslant \log d \log\log d$ will yield the best upper bound on $\theta(s, d)$ obtainable from this construction. One would like to choose d as a base to make the upper bound equal the lower bound, but this can only be done when $s = 1$.

We present one further construction useful when d is large. In particular, if $d = (s - 1)\binom{m-1}{s-2}$ for some m, then $\theta(s, d) \leqslant (s - 1)/d$. Call integers of the form $(s - 1)\binom{m-1}{s-2}$ *special numbers*. Since $\theta(s, d)$ is decreasing in d, this yields $\theta(s, d) \leqslant (s - 1)/q$, where q is the largest special number less than or equal to d. Finally, Section IV returns to small cases to mention some numerical results from a linear programming formulation.

II. Relation to Previous Work

The constructions for fixed s and d have the ratio m/n going to 1 as s grows. However, related to this problem is an important and well-studied problem in which $\theta(n, s, d)$ is of interest for a fixed value of $\alpha = s/n$ rather than fixed s. Work in this area began with graphs called *concentrators*, first studied by Pinsker [10] and Margulis [9]. Variants of this problem include the design of connectors [12], nonblocking networks [5], [6], generalized connectors [12], and expanders [1], [2], [8], [13]. More clearly related to our problem is the subsequent *$(n, \theta, k, \alpha, c)$-bounded strong concentrator* defined by Gabber and Galil [8] to be a bipartite graph with n inputs, θn outputs, and at most kn edges, such that any set S of at most αn inputs has at least $c|S|$ neighbors. (Our graphs are $(n, m/n, dm/n, s/n, 1)$-bounded strong concentrators.) These graphs can be used in a recursive construction of *superconcentrators*, originally defined by Valiant [14], which are networks with n inputs and n outputs that have vertex-disjoint paths from any set of inputs to any set of outputs of the same size. One wishes to construct these graphs, which have many applications, with a small number of edges. One can construct an n-superconcentrator from an $(n, k, t, a, 1)$-bounded strong concentrator and a θn superconcentrator [11], hence the desire to minimize θ. Increasing α allows saving some edges in the recursive construction.

We have no new constructions to present for this situation; we merely cite results from the theory of concentrators that apply here. The simplest and most interesting case is $\alpha = s/n = 1/2$, treated by Pippenger [11].

Lemma [11]: For every m there exists a bipartite graph with $6m$ inputs and $4m$ outputs in which every input has degree at most 6, every output has degree at most 9, and every set of at most $3m$ outputs is matchable.

In other words, $\theta(6m, 3m, 9) \leqslant 2/3$. The surprise here is that for $d = 9$, $\theta(n, \alpha n, d)$ does not go to 1 as $n \to \infty$, whereas we have proved that for $d = 2$ it goes to 1. Chung [6] later obtained a more general result which is not restricted to a single value of d, and which showed that, for sufficiently small α and sufficiently large n, $\theta(n, \alpha n, d) \leqslant 5/6$ when $d = 6$.

Lemma [6]: Let m, t, a, and b be integers with $a \geqslant b \geqslant 2$, and choose a real number $0 < \alpha < 1$. If m is sufficiently large and the conditions below hold, then there exists a bipartite graph with $n = am$ inputs, bm outputs, input degree bt, output degree $d = at$, and every set of at most $s = \alpha n$ inputs matchable. Let $H(z)$ be the entropy function $-z \log z - (1 - z) \log (1 - z)$. The conditions are

$$t > \frac{H(\alpha) + (b/a)H(a\alpha/b)}{bH(\alpha) - \alpha a H(b/a)} \qquad \alpha < \frac{b(b-2)}{ab - a - b} \qquad bt > 4.$$

It must be noted that these are nonconstructive results obtained from counting arguments. It is important to discuss some of the constructive results in this related area as well for other values of d.

Margulis [9] described a family of explicit bipartite graphs H which consist of an ordered pair (A, B) of sets of nodes. Let m be an integer greater than 1 and let Z_m be the ring of residues modulo m. Let A_m and B_m be $Z_m \times Z_m$. Each element (x, y) of A_m is connected to the following five elements of B_m: a) (x, y), b) $(x + 1, y)$, c) $(x, y + 1)$, d) $(x + y, y)$, e) $(-y, x)$. Margulis showed by the theory of group representations that there exists a constant $g > 0$ such that for $n = m^2$ and $m = 1, 2, \cdots$, H_n is an $(n, 5, g)$ expander. (An (n, k, c) expander is a k-regular bipartite graph on the set of n input and n output vertices such that every set of $x \leqslant n/2$ inputs is joined by edges to at least $x + c(1 - x/n)x$ different outputs.)

Angluin [4] proposed an extension of the construction of Margulis by using three instead of five linear transformations of $Z_m \times Z_m$: a) (x, y), b) $(x + y, y)$, c) $(y + 1, -x)$. This construction yielded $(n, 3, g)$ expanders.

Alon *et al.* [3] gave the construction of (n, k, g) expanders for $k = 4, 6, 8, 12$ by using k linear transformations on Z_m similar to Margulis.

In addition, Alon *et al.* [2] proposed the construction of $(n, k + 1, g)$ expanders from k-regular Cayley graphs on n vertices using the concept of double covers of graphs.

Gabber and Galil [8] gave explicit constructions of bounded strong concentrators from linear expanders. Let p be a fixed natural number, and n be a natural number such that $np/(p + 1)$ is an even square. They constructed a bipartite graph with n inputs and $np/(p + 1)$ outputs. The inputs are partitioned into two disjoint parts, the big one of size $np/(p + 1)$ and the small one of size $l = n/(p + 1)$. The big part and the outputs are connected by an $(np/(p + 1), k, 2/(p - 1))$ expander. Every output is connected to exactly one input in the small part. For every $j = 0, 1, \cdots p - 1$, outputs numbered $jl + 1, jl + 2, \cdots, (j + 1)l$ are connected to inputs numbered $1, 2, \cdots, l$ in the small part. Gabber and Galil showed that the resultant graph is an $(n, p/(p + 1), k', 1/2)$-bounded concentrator, where $k' = (k + 1)p/(p + 1)$. In our context,

$$\theta(n, s, d) = p/(p+1) < 1.$$

From the above results, we make some observations about our problem of constructing bipartite graphs with minimum ratio of output to input nodes $\theta(n, s, d)$ satisfying bounded degree d of the output nodes, and the condition that each set of s inputs is matchable to s distinct outputs. We can conclude that $\theta(n, s, d) < 1$ from Alon's result for $d = 13$, from Pippenger's result for $d = 9$, Alon's result for $d = 8$, $d = 7$, from Chung's and Margulis' result for $d = 6$, Alon's result for $d = 5$, Angluin's result for $d = 4$. Also, from the existence of general k-regular expanders from Alon's result [2] combined with the construction of concentrators from [8] which have degree of output nodes $= k + 1$, we can conclude that $\theta(n, s, d) < 1$

for all $d = k + 1$ for which k-regular expanders exist. Angluin [4] has shown that Margulis' [9] construction of linear expanders from linear transformations in Z_m is not valid for $k = 2$. Hence, we cannot say anything about the value of $\theta(n, s, d)$ for $d = 3$ from the above results.

In this correspondence, we give some general constructions of bipartite graphs satisfying the s-matching condition for a fixed value of s rather than a fixed value of $\alpha = s/n$ which was done by other researchers.

III. Small Values of S

In this section, we focus on explicit constructions, which are useful when s is not too large. We begin with a construction that is not very good in general, but is valid for all (s, d) and is optimal when $s \leqslant 2$ or $d \leqslant 2$.

Theorem 1: $\theta(s, d) \leqslant s/(s + d - 1)$, with equality when $s \leqslant 2$ or $d \leqslant 2$.

Proof. The construction. By the remark in the Introduction, it suffices to provide a component with $s + d - 1$ inputs and s outputs. Construct such a component by joining output y_j to inputs $x_j, \cdots, x_{j+d-1}$. This component has the (s, d)-matching property, because the matching $M = \{x_i y_i: 1 \leqslant i \leqslant d\}$ can be altered iteratively to satisfy any set S of at most s inputs as follows. For $i = 1, \cdots, d - 1$, if $x_i S$ and x_i is the $t + 1$th input missed by S, alter the current M by replacing $\{x_j y_{j-t}: i \leqslant j \leqslant d + t\}$ by $\{x_{j+1} y_{j-t}: i \leqslant j \leqslant d + t\}$.

Optimality for $s \leqslant 2$, i.e., $\theta(n, 2, d) \geqslant 2/(d + 1)$ (obvious for $s = 1$): Consider a component G of an optimal graph on n inputs. G has k inputs and $m = \theta k$ outputs. Let t be the number of inputs in G having degree 1; call them *leaves*. By the pigeonhole principle, $m \geqslant t$, else there is an unsatisfied pair of leaves. This means showing $t \geqslant 2k/(d + 1)$ will complete the proof. If $t < 2k/(d + 1)$, we get a contradiction by counting edges of G. Counting them by inputs yields at least $t + 2(k - t) = 2k - t > 2kd/(d + 1)$. Counting them by outputs yields at most $dm = d\theta k$. Together these imply $\theta \geqslant 2/(d + 1)$.

Optimality for $d = 2$, i.e., $\theta(n, s, 2) \geqslant s/(s + 1)$ (obvious for $d = 1$): Again, consider an optimal component G with k inputs and $m = \theta k$ outputs. We may assume $k > s$, else $\theta \geqslant 1$. Again we count edges. Since the component is connected, there must be at least $m + k - 1$, but there are at most $2m$. Hence, $m \geqslant k - 1$, or $\theta \geqslant (k - 1)/k \geqslant s/(s + 1)$. □

Example 1: We give an example construction for $s = 4$, $d = 3$. Then there are six input nodes and four output nodes. Output node 1 is connected to input nodes 1, 2, 3; output node 2 is connected to 2, 3, 4; output node 3 is connected to inputs 3, 4, 5; output node 4 is connected to inputs 4, 5, 6.

Now we suppose d is somewhat larger, but we prevent s from growing arbitrarily in relation to d, i.e., $s \leqslant d^r$. For this case, we provide only a construction. Ignoring the ceiling brackets may make it look essentially optimal, since the degree restriction places a trivial lower bound of $1/d$ on $\theta(s, d)$ and $s^{1/r}$ can be made arbitrarily small by letting r grow, but $\lceil s^{1/r}/r \rceil$ cannot be less than 1, and there is certainly no further improvement after reaching that point.

Theorem 2: For $s \leqslant d^r$, $\theta(d^r, s, d) \leqslant r\lceil s^{1/r}/r \rceil/d$.

Proof: We construct a component using d^r inputs and $rd^{r-1}\lceil s^{1/r}/r \rceil$ outputs. Arrange the inputs in an r-dimensional array with side d. Assign an output to the d inputs in each one-dimensional row of the array, i.e., fixing $r - 1$ coordinates. We use $\lceil s^{1/r}/r \rceil$ copies of this set of rd^{r-1} outputs. The outputs all have degree d; we need only verify that every set of at most s inputs is satisfied.

Each input neighbors r outputs in each copy. Given a set of $k \leqslant s$ inputs, suppose its positions in the array meet $p = \Sigma p_i$ outputs in each copy, p_i in the ith direction. Then it has $p\lceil s^{1/r}/r \rceil$ output neighbors. For a given k, p is minimized when the positions are arranged in an r-dimensional cube of side $k^{(r-1)/r}$. In particular, $p \geqslant rk^{(r-1)/r}$. This implies $p\lceil s^{1/r}/r \rceil \geqslant k$, as desired. □

Example 2: We show a construction for $d = 4$, $r = 3$, and $s = 8$. We consider a three-dimensional array of input nodes, with each side of the cube having four nodes. Hence, the array represents 64 nodes. We define $3 \times 16 \times \lceil 2/3 \rceil = 48$ output nodes. Each output node is connected to four input nodes in each one-dimensional row of the array. The outputs have degree 4.

Remarks: 1) We can do a bit better by not using identical copies of the outputs, but rather distributing them differently over the coordinates in each copy. For example, in the jth copy, let the input sets covered by an output be a point x and all translates of x mod d by $(t_{1,j}, \cdots, t_{r,j})$, where t_{ij} is 0 or relatively prime to d. By appropriate choice of the t_{ij}, an s set configured to hit the minimum number of outputs in one copy will hit more than the minimum in other copies. In other words, fewer copies will be needed. Similarly, we can shave a little off the outputs in each copy when $s^{1/r}/r$ is just a little more than an integer.

2) The fact that this construction is valid for an arbitrary number of dimensions suggests that it may be possible to improve on the *expander graphs* of [8] and [9] by generalizing their constructions to higher dimensions. Also, the expander graphs themselves may yield better constructions for the graphs studied here.

Finally, since this construction only yields $\theta(s, d) \leqslant \log d/d$ for large d and fixed s, we present another that yields about $(s - 1)/d$. As noted in the Introduction, we get $\theta(s, d) \leqslant (s - 1)/q$, where q is the largest special number less than or equal to d.

Theorem 3: If $d = (s - 1)\binom{m-1}{s-2}$, then $\theta(s, d) \leqslant (s - 1)/d$.

Proof: We construct a component with m outputs and $n = (s - 1)\binom{m}{s-1}$ inputs. Partition the inputs into $\binom{m}{s-1}$ disjoint classes of $s - 1$ inputs each. Put these classes in 1-1 correspondence with the subsets of size $s - 1$ of the outputs. For each output set of size $s - 1$, join each of its vertices to each of the vertices in the corresponding class of $s - 1$ inputs.

Any set of inputs in a single class has size at most $s - 1$ and has $s - 1$ output neighbors. On the other hand, any set of inputs intersecting more than one class has at least two distinct sets of $s - 1$ neighbors, hence at least s neighbors altogether. Furthermore, since each output belongs to $\binom{m-1}{s-2}$ sets of size $s - 1$, each output has degree $(s - 1)\binom{m-1}{s-2}$. Finally, $m/n = m/(s - 1)\binom{m}{s-1} = m/m\binom{m-1}{s-2} = (s - 1)/d$. □

Example 3: We now give an example construction for $m = 4$ outputs and $n = 12$ inputs for $s = 3$. Partition the inputs into six disjoint classes of two each: {1, 2}, {3, 4}, {5, 6}, {7, 8}, {9, 10}, {11, 12}. Put these classes into 1-1 correspondence with the subsets of size 2 of the outputs 1–4 (there are six such subsets: {1, 2}, {1, 3}, {1, 4}, {2, 3}; {2, 4}, {3, 4}). For each output set of size 2, join each of its vertices to each of the vertices of the corresponding class of two inputs. Specifically, we connect output node 1 to input nodes 1, 2, 3, 4, 5, 6; output node 2 to input nodes 1, 2, 7, 8, 9, 10; output node 3 to input nodes 3, 4, 7, 8, 11, 12; and output node 4 to input nodes 5, 6, 9, 10, 11, 12. Each output node has degree 6.

IV. Integer Programming Formulation

The constraints and objective of the (s, d)-matching problem can be formulated as a 0, 1-integer programming problem with linear constraints. It is a "multicovering" problem; the dual is a weighted packing problem.

To define the program, note that we may assume all the outputs have degree d, since extra edges thrown in cannot hurt. In running the programs, we have considered only the class of graphs where no output nodes share the same neighborhoods. We have not proved that the optimal configuration is of this type. We associate with each potential output one of the d subsets of the n inputs. We want to choose the minimum number of these that will satisfy the constraints.

The constraint matrix is the incidence matrix between the d subsets of the inputs (corresponding to possible output neighborhoods) and the subsets of size at most s of the inputs. The constraint for a set of size k is that it intersect at least k of the chosen output neighborhoods. Hence, the constraints look like $\Sigma_i x_i a_i$, $s \geqslant |S|$, and the objective function is min Σx_i.

In Table I, we show the optimal number of outputs for some small values of n, with s and d varying from 3 to 5. These are the results of an integer linear program run using a Multipurpose Optimization

TABLE I
LINEAR PROGRAMMING RESULTS. ENTRIES IN THE TABLE SHOW THE OPTIMAL NUMBER OF OUTPUTS AND $\theta(n, s, d)$ REPRESENTED AS (m, θ) FOR VARIOUS VALUES OF n, s, AND d

s, d \ n	5	6	7	8	9	10
$s = 3, d = 3$	3, 0.600	4, 0.677	4, 0.571	5, 0.625	5, 0.556	6, 0.600
$s = 3, d = 4$	3, 0.600	3, 0.500	4, 0.571	4, 0.500	5, 0.556	
$s = 3, d = 5$	3, 0.600	3, 0.500	3, 0.429	4, 0.500		
$s = 4, d = 3$	4, 0.800	4, 0.667	5, 0.714	5, 0.625		
$s = 4, d = 4$	4, 0.800	4, 0.667	4, 0.571	5, 0.625		
$s = 4, d = 5$	4, 0.800	4, 0.667	4, 0.571	4, 0.500		
$s = 5, d = 3$	5, 1.000	5, 0.883	5, 0.714	6, 0.750		
$s = 5, d = 4$	5, 1.000	5, 0.883	5, 0.714	5, 0.625		
$s = 5, d = 4$	5, 1.000	5, 0.883	5, 0.714			

System Package [7] running under the NOS operating system on a CDC Cyber-175 computer. Larger problems than the ones listed could not be run due to limited memory space. The columns are indexed by the number of inputs n, and the entries give the minimum m and the minimum output-input ratio m/n.

V. SUMMARY OF RESULTS

In this correspondence, we have treated the problem of minimizing the number of service ports of a central facility which serves a number of users. At any time, a set of at most s users may want to use the facility, and one user can be connected to each port at a given time. We assume there are direct communication links from users to service ports, with at most d links incident at a single service port. This constraint on degree may model a fan-in limitation of some circuitry at the service port. The problem is to satisfy the constraints with the minimum number of service ports. We have given a graph-theoretic formulation of the problem in this correspondence. Given a set of n "input" nodes, we defined $f(n, s, d)$ to be the minimum number of "output" nodes in a bipartite graph such that every set of at most s inputs is joined to at least s outputs, subject to the restriction that every output node has degree at most d. We then defined $\theta(s, d) = \lim_{n\to\infty} f(n, s, d)$. A general construction yielded $\theta(s, d) \leqslant s/(s + d - 1)$, and this was shown to be optimal when $s \leqslant 2$ or $d \leqslant 2$. For larger values of s and d, better constructions were given. When $s \leqslant d^r$, then we showed that $\theta(s, d) \leqslant r\lceil s^{1/r}/r \rceil/d$. For d fixed and s large, the problem is closely related to the construction of bounded concentrators and expander graphs, used in VLSI applications, and results there can be applied. A linear programming formulation was given.

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A Data Compression Technique for Built-In Self-Test

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Abstract—**Data compression is often used to reduce the complexity of test data in the area of fault diagnosis in digital systems. A data compression technique called self-testable and error-propagating space compression is proposed and analyzed. Faults in a realization of Exclusive-OR and Exclusive-NOR gates are analyzed and the use of these gates in the design of self-testing and error-propagating space compres-**

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