MATH 247 HONORS, FALL 1999 - PROBLEM SET 9

Test #2 on Wednesday, October 27, 7:30-9:30PM, 141 Altgeld Hall.

WARMUP PROBLEMS: 8.24, 9.9, 9.19. Do not write these up. These are easier problems to check understanding of the material.

WRITTEN PROBLEMS. Do five of the following six problems; full credit requires complete justifications in sentences. Due Friday, Oct. 29, because of test on Wednesday. Problem set #10 due Wednesday, Nov. 3.

1. Determine when the sum of two Pythagorean triples (under componentwise addition) is a Pythagorean triple. (Hint: Several methods work. 1) Compare with Exercise 1.19, or 2) Find equations for one triple in terms of the other, or 3) Place the three triples on circles in the plane and argue geometrically.)

2. *Alternative proof of characterization of Pythagorean triples.* This exercise develops an alternative proof that every Pythagorean triple has the form described in Theorem 8.22. Let (a, b, c) be a Pythagorean triple such that a, b, c have no common factor (thus $\gcd(a, b) = \gcd(b, c) = \gcd(a, c) = 1$).

a) Prove that exactly one of a and b is even.

b) Let a be the even member of $\{a, b\}$. Prove that $(c+b)/2$ and $(c-b)/2$ are relatively prime and are squares of integers.

c) Given the result of part (b), let $(c + b)/2 = z^2$ and $(c - b)/2 = y^2$. Prove that $a = 2yz$, $b = z^2 - y^2$, and $c = z^2 + y^2$.

(Comment: the proof of Theorem 8.22 in the text emphasizes geometry and the properties of rational numbers. This proof emphasizes divisibility and primes.)

3. From n equally spaced points on a circle, a triple of three distinct points is chosen at random. What is the probability that they form an equilateral triangle? An isosceles triangle? A triangle with sides of distinct lengths?

4. In Bertrand's Ballot Problem, suppose the outcome is (a, b), with $a > b$, and the votes are counted in random order. What is the probability that A is always ahead of B? What is the probability that the score is tied at some point after the beginning?

5. Half the females and one-third of the males in a class are smokers. Also, two-thirds of the students are male. What fraction of the smokers are female?

6. Beginning with A, players A and B alternate flipping a coin that has probability p of showing heads. The first player to get heads wins. Let x be the probability that A wins. Determine x as a function of p. Evaluate the formula in the special case of a fair coin, $p = .5$. (Hint: use conditional probability to obtain an equation for x.)

PROBLEMS FOR CLASS DISCUSSION

Pair 1	Pair 2	Pair 3
9.12, 9.38	9.11, 9.37	8.26, 9.17