MATH 247 HONORS, FALL 1999 - PROBLEM SET 8

No class on Friday, October 15.

WARMUP PROBLEMS: 7.38, 8.10, 8.17. Do not write these up. These are easier problems to check understanding of the material.

WRITTEN PROBLEMS. Do five of the following six problems. Due Wednesday, Oct. 20. Full credit requires complete justifications in sentences.

1. The base 10 representation of an integer is *palindromic* if the digits read the same when written forward or backward. Prove that every palindromic integer with an even number of digits is divisible by 11. More generally, prove that every integer whose base k representation is palindromic and has even length is divisible by $k+1$.

2. *Test for divisibility by 7.*

a) Let $a_k \cdots a_0$ be the base 10 representation of n. We can determine whether n is divisible by 7 by treating n as $\sum a_i 10^i$ and reducing the powers of 10 modulo 7; we have discussed this approach to divisibility by 9. Apply this to check whether 7 divides 535801.

b) Given a positive integer n, let $f(n)$ be the integer formed by subtracting twice the last base 10 digit of n from the number formed by the remaining digits of n. For example, if $n = 154$, then $f(n) = 15 - 8 = 7$. Prove that $7|n$ if and only if $7|f(n)$. Apply this to check whether 7 divides 535801. (Hint: To prove that $7|n$ if and only if $7|f(n)$, prove first that $7|n$ if and only if $7|[10f(n)]$.)

3. 1500 soldiers arrive in training camp. A few soldiers desert the camp. The drill sergeants divide the remaining soldiers into groups of five and discover that there is 1 left over. When they divide them into groups of 7, there are 3 left over, and when they divide them into groups of 11, there are again 3 left over. Determine the number of deserters.

4. By Lemma 7.20, $\{a, 2a, \ldots, (p-1)a\}$ have distinct remainders modulo p when a and p are relatively prime. Use this to give a short proof of Fermat's Little Theorem.

5. Let a/m and b/n be rational numbers expressed in lowest terms. Prove that $(an + bm)/(mn)$ is in lowest terms if and only if m and n are relatively prime.

6. In the Billiard Problem (Solution 8.10), for each corner of the square determine the condition on the slope s so that the process ends there.

PROBLEMS FOR CLASS DISCUSSION

Solve these before the class discussion. If stumped, come to ask questions.

Group	Pair 1	Pair 2	Pair 3
Orange	Cuckler	McFerrin/Kueker	Schmitt/Toledo
Blue	Smyth	Fast/Sanzo	Wolak
Green	Scheiwe	Radebaugh/Shaftman	Barker/Chen
	7.29, 8.19	7.32, 7.44	7.40, 8.11