MATH 247 HONORS, FALL 1999 - PROBLEM SET 7

WARMUP PROBLEMS: 6.40, 7.9, 7.11, 7.16. Do not write these up. Just think about how to solve them to make sure you understand the material before working on the written homework.

WRITTEN PROBLEMS. Do five of the following six problems. Due Wednesday, Oct. 13. Full credit requires complete justifications in sentences.

1. The royal treasury has 500 7-ounce weights, 500 13-ounce weights, and a balance scale. An envoy arrives with a bar of gold, claiming it weighs 500 ounces. Can the treasury determine whether the envoy is lying? If so, how? What if the weights are six-ounce and nine-ounce weights?

2. The *least common multiple (lcm)* of natural numbers a and b is the least natural number divisible by both. Prove that $\text{lcm}\,(a, b) \cdot \gcd(a, b) = a \cdot b$.

3. A bear's cage has two jars of jelly beans, one with x beans and the other with y. Each jar has a lever. When a jar has at least two beans, pressing its lever will give the bear one bean from it and move one bean from it to the other jar; otherwise the lever has no effect. Obtain necessary and sufficient conditions on the pair x, y so that the bear can eat all the beans except one.

4. Prove that every year (including leap years) has at least one Friday the 13th. What is the maximum number of Friday the 13ths in a year?

5. Suppose $m, n, p \in \mathbb{Z}$ and 5 divides $m^2 + n^2 + p^2$. Prove that 5 divides at least one of $\{m, n, p\}$.

6. Prove that if two natural numbers have the same number of copies of each digit in their decimal representations, then they differ by a multiple of 9.

PROBLEMS FOR CLASS DISCUSSION

Solve your group problems before the class discussion. If you are stumped, come to ask questions.

Pair 1 - 6.33, 7.28.
Pair 2 - 6.46, 7.6.
Pair 3 - 6.48, 7.13.