

MATH 247 HONORS, FALL 1999 - PROBLEM SET 5

Do five of the following six problems. Due Friday, Oct. 1, because of test on Wednesday. Test #1 Wednesday, Sept. 29, 7:30-9:30PM, 241 Altgeld Hall.

1. Let A_1, A_2, \dots be a sequence of sets, each of which is countable. Prove that the union of all the sets in the sequence is a countable set.
2. Construct an explicit bijection from the open interval $(0, 1)$ to the closed interval $[0, 1]$.
3. Consider a dial having a pointer that is equally likely to point to each of n regions numbered $1, 2, \dots, n$. When we spin the dial three times, what is the probability that the sum of the selected numbers is n ?
4. There are 999,999 natural numbers less than one million. For $1 \leq k \leq 6$, determine how many of these numbers have k distinct numbers in their decimal representations, counting leading zeros. In other words, treat 111 as 000111 and count it for $k = 2$.
5. Use Pascal's Formula to prove the binomial theorem by induction on n .
6. *Summing the cubes.*
 - a) Prove directly that $m^3 = 6\binom{m}{3} + 6\binom{m}{2} + m$.
 - b) Use part (a) to prove that $\sum_{i=1}^n i^3 = \left(\frac{n(n+1)}{2}\right)^2$ (without using induction).
 - c) Prove part (a) by counting a set in two ways. (Hint: count the 3-tuples that can be formed from $[m]$.)

PROBLEMS FOR CLASS DISCUSSION

Since we discuss permutations in Chapter 5, with this assignment we permute the groups. Each pair consists of person A and person B; decide which of you is which. Person A from pair i stays in pair i . Person B from pair 1 moves to pair 2, from pair 2 moves to pair 3, and from pair 3 moves to pair 1. When a pair has only one person, it is Person A.

After doing that, the new pair 1 stays in its color group. Orange pair 2 now becomes blue 2, blue 2 becomes green 2, and green 2 becomes orange 2. Orange pair 3 becomes green 3, green 3 becomes blue 3, and blue 3 becomes orange 3.

Problems for new pairs:

Pair 1 - 4.39, 5.20, 5.30.

Pair 2 - 5.6, 5.18, 5.34.

Pair 3 - 5.19, 5.26, 5.40.