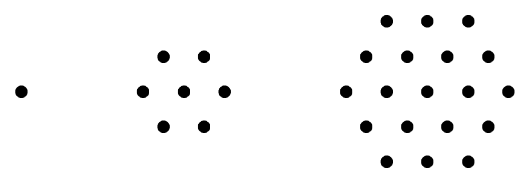
MATH 247 HONORS, FALL 1999 - PROBLEM SET 3

Do five of the following six problems. Due Wednesday, Sep. 15.

1. Let S_n be the hexagonal arrangement consisting of n rings of dots, as illustrated below for $n \in \{1, 2, 3\}$. Let a_n be the number of dots in S_n. Compute a_n. Compute $\sum_{k=1}^{n} a_k$.



2. Suppose $f\colon \mathbb{R} \to \mathbb{R}$ satisfies $f(xy) = xf(y) + yf(x)$ for all $x, y \in \mathbb{R}$. Prove that $f(1) = 0$ and that $f(u^n) = nu^{n-1}f(u)$ for all $n \in \mathbb{N}$ and $u \in \mathbb{R}$.

3. Determine the set of positive real numbers x such that the inequality $x^n + x < x^{n+1}$ holds for all $n \in \mathbb{N}$.

4. *L-tilings.* Prove that R has an L-tiling in the following situations.
a) R is a 2^k by 2^k chessboard with one corner square removed.
b) R is a 2^k by 2^k chessboard with *any* single square removed.

5. Consider a row of n boxes, each containing a number, such that the number in the ith box is the ith smallest number. Given a number x, one would like to know whether x appears in one of the boxes. Iteratively, one can look at the number in a box and then decide what box to look in next.

a) Prove that when $n < 2^k$, there is a strategy that always determines whether x is present by looking in at most k boxes, no matter what x is or what numbers are in the boxes.

b) Prove that when $n \geq 2^k$, there is no strategy that will always answer the question by looking in at most k boxes.

6. *The December 31 Game.* Two players alternately name dates. On each move, a player can increase the month or the day of the month but not both. The starting position is January 1, and the player who names December 31 wins. According to the rules, the first player can start by naming some day in January after the first or the first of some month after January. For example, (Jan. 5, Mar. 5, Mar. 15, Apr. 15, Apr. 25, Nov. 25, Nov. 30, Dec. 30, Dec. 31) is an instance of the game won by the first player. Derive a winning strategy for the first player. (Hint: use strong induction to describe the "winning dates".)

PROBLEMS FOR CLASS DISCUSION - CHAPTER 3

Pair 1 - 3.20, 3.39, 3.47.
Pair 2 - 3.24, 3.38, 3.50.
Pair 3 - 3.26, 3.34, 3.53.