

MATH 247 HONORS, FALL 1999 - PROBLEM SET 12

WARMUP PROBLEMS: 13.30, 13.31, 14.2, 14.6. Do not write these up. These are easier problems to check understanding of the material.

WRITTEN PROBLEMS. Do five of the following six problems; full credit requires complete justifications in sentences. Due Wednesday, Nov. 17.

1. Prove that a real number has more than one k -ary expansion if and only if it is expressible as a fraction using a denominator that is a power of k . (Recall that the k -ary expansions correspond to the k -ary sequences $\langle c \rangle$.)
2. Let a and b be natural numbers, expressed in decimal expansion.
 - a) Prove that long division of a by b yields the decimal expansion of a/b .
 - b) Use the pigeonhole principle and long division to prove that the decimal expansion of a/b has a period of length less than b .
3. Let $\langle a \rangle$ and $\langle b \rangle$ be sequences, with $b_n \neq 0$ for all n . Prove that if $a_n \rightarrow L$ and $b_n \rightarrow M \neq 0$, then $a_n/b_n \rightarrow L/M$. (Hint: First prove it when $a_n = 1$ for all n .)
4. Suppose that $x_1 > -1$ and that $x_{n+1} = \sqrt{1 + x_n}$ for $n \geq 1$. Prove that $\lim_{n \rightarrow \infty} x_n$ exists, and find the limit.
5. Let $\langle x \rangle$ be a sequence satisfying the recurrence $x_{n+1} = x_n^2 - 4x_n + 6$.
 - a) If $\lim_{n \rightarrow \infty} x_n$ exists and equals L , what possible values can L have?
 - b) The behavior of x_n as $n \rightarrow \infty$ depends on the initial value x_0 . For each $x_0 \in \mathbb{R}$, describe this behavior. (Hint: Graph the functions defined by x and $x^2 - 4x + 6$ and interpret the graphs, or obtain a recurrence for the sequence $\{y_n\}$ defined by $y_n = x_n - 2$ and study its behavior.)
6. For $c > 0$, let $x_n = (c^n + 1)^{1/n}$. Determine $\lim_{n \rightarrow \infty} x_n$. More generally, find $\lim_{n \rightarrow \infty} (a^n + b^n)^{1/n}$. (Hint: Consider $c < 1$ first and use the Squeeze Theorem.)

PROBLEMS FOR CLASS DISCUSSION

Please solve your group problems before the class discussion! If stumped, come to ask questions.

Pair 1	Pair 2	Pair 3
13.36, 14.24	14.8, 14.11	13.35, 14.22