

# MATH 247 HONORS, FALL 1999 - PROBLEM SET 11

WARMUP PROBLEMS: 10.27, 10.32, 13.17, 13.19. Do not write these up. These are easier problems to check understanding of the material.

WRITTEN PROBLEMS. Do five of the following six problems; full credit requires complete justifications in sentences. Due Wednesday, Nov. 10.

1. Let  $\phi(m)$  denote the Euler totient function (the number of elements of  $[m]$  that are relatively prime to  $m$ ). If  $p, q$  are distinct prime numbers, prove that  $\phi(pq) = \phi(p)\phi(q)$ . In general, when  $P(m)$  denotes the set of distinct prime factors of  $m$ , prove that  $\phi(m) = m \prod_{p \in P(m)} (1 - 1/p)$ .

2. A math department has  $n$  professors and  $2n$  courses, each professor teaching two courses each semester. How many ways are there to assign the courses in the fall semester? How many ways are there to assign the courses in the spring semester such that no professor teaches the same pair of courses in the spring as in the fall? If all the assignments are equally likely, what is the probability of this event?

3. Given the five types of coins (pennies, nickels, dimes, quarters, half-dollars), how many ways can one select  $n$  coins so that no coin is selected more than 4 times? (Hint: Use inclusion-exclusion and selections with repetition.)

4. *Limits, using the  $\epsilon$ - $N$  definition.*

a) Use the definition of limit to prove that  $\lim \sqrt{1 + n^{-1}} = 1$ .

b) Use the definition of limit to prove that  $\lim[(1 + a_n)^{-1}] = \frac{1}{2}$  if  $\lim a_n = 1$ .

5. Let  $x_n = (1 + n)/(1 + 2n)$ . Prove that  $\lim_{n \rightarrow \infty} x_n$  exists by using Monotone Convergence. Prove that  $\lim_{n \rightarrow \infty} x_n = 1/2$  by using the definition of limit.

6. Let  $x_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n}$ . Prove that  $\lim_{n \rightarrow \infty} x_n$  exists. (Comment: In fact, the limit equals  $\ln 2$ , but this information is not needed for this exercise.)

## PROBLEMS FOR CLASS DISCUSSION

Pair 1	Pair 2	Pair 3
10.38, 13.28	10.36, 13.20	10.29, 13.24