MATH 247 HONORS, FALL 1999 - PROBLEM SET 10

Test #2 on Wednesday, October 27, 7:30-9:30PM, 141 Altgeld Hall.

WARMUP PROBLEMS: 9.28, 9.33, 10.2, 10.3. Do not write these up. These are easier problems to check understanding of the material.

WRITTEN PROBLEMS. Do five of the following six problems; full credit requires complete justifications in sentences. Due Wednesday, Nov. 3.

1. Let X be a random variable that takes values only in $[n]$. Prove that $E(X) = \sum_{k=1}^{n} \text{Prob}\,(X \geq k)$.

2. Suppose that n pairs of socks are put into the laundry, with each sock having one mate. The laundry machine randomly eats socks; a random set of k socks returns. Determine the expected number of complete pairs of returned socks. (Hint: Use the linearity of expectation.)

3. We have six dice; faces are equally likely to appear when a die is rolled. Each die has three red faces, two green faces, and one blue face. We roll the six dice. Derive a formula for the probability that we get a red face on three dice, a green face on two dice, and a blue face on one. (Hint: The answer reduces to a fraction with denominator 36.)

4. Prove that every set of seven distinct integers contains a pair whose sum or difference is a multiple of 10.

5. The numbers 1 through 12 have fallen off the face of a clock and have been replaced in some random order. Prove that some set of three consecutive numbers has sum at least 20. Prove that some set of five consecutive numbers has sum at least 33. For three consecutive numbers, use more detailed analysis to determine whether it is possible for all the sums to be 19 or 20.

6. The *fractional part* of x is the amount by which it exceeds $\lfloor x \rfloor$. For $x \in \mathbb{R}$ and $n \in \mathbb{N}$, let $S = \{x, 2x, \ldots, (n-1)x\}$.

a) Prove that if some pair of numbers in S have fractional parts that differ by at most $1/n$, then some number in S is within $1/n$ of an integer.

b) Use part (a) to prove that some number in S is within $1/n$ of an integer.

PROBLEMS FOR CLASS DISCUSSION

Pair 1	Pair 2	Pair 3
9.34, 10.19	9.22, 10.16	9.25, 10.8