Math 347 C1
FINAL EXAM
4 August 2006

NAME:_____________________________________
ID:_____________________________________

INSTRUCTIONS:
1. Do each problem.
2. Show all work.
3. No books, no notes, no calculators.
4. Each problem is worth the same number of points.

Score
1. _________
2. _________
3. _________
4. _________
5. _________
6. _________
7. _________
8. _________

TOTAL:_________
1. How many solutions in natural numbers are there for:

\[ x_1 + x_2 + x_3 + x_4 = 50. \]
2. Find the coefficients of $x^5y^6$ and $x^3y^8$ in $(x + y)^{11}$. 
3. Solve: \( a_n = 2a_{n-1} + 3a_{n-2}, \ a_0 = a_1 = 1. \)
4. Prove by induction:
\[
\sum_{k=1}^{n} (2k + 1) = n^2 + 2n.
\]
5. Show that the function $f : R \to R$ given by $f(x) = \frac{x}{1+x^2}$ is not injective.
   (Hint: Divide numerator and denominator by $x^2$.)
6. Suppose that \( \lim_{x \to 0} f(x) = 0 \). Show that there is a sequence \( \langle x_n \rangle \), such that 
\[ |f(x_n)| < \frac{1}{n}. \]
7. Show that the congruence \( x^2 \equiv 2 \pmod{5} \) has no solution in \( N \).
8. Let $P$ be the set of all people.
   Let $T$ be the set of all times.
   Let $f(x, t)$ mean that you can fool person $x$ at time $t$.

Consider the two statements.

i) You can fool all of the people all of the time.

ii) You can never fool anyone.

Express the two statements using $P$, $T$, $f$ and quantifiers. Is either statement the negation of the other? Explain.