Polynomials

A *polynomial in one variable* is a function in which the variable is only to whole number powers, and the variable does not appear in denominators, in exponents, under radicals, or in between absolute value signs or greatest integer signs.

Examples of polynomials in one variable:

\[-3x^4 + x^3 - x\sqrt{4} + 8\]
\[1 - \frac{3}{5}t^7\]
\[(x^2 + x + 1)(3x - 8)\]
\[3\]

Examples of expressions that are not polynomials:

\[3x^2 - \sqrt{x}\]
\[2^x\]
\[x^\pi\]
\[\frac{x + 1}{3x^4 - 1}\]

Degrees of Polynomials

The *degree* of a polynomial is the highest power of the variable that occurs. Remember that an expression that does not contain a variable is a polynomial, in which case the degree is zero.

**Exercise:** Determine the degrees of the above expressions that are polynomials.

Leading Coefficients of Polynomials

The *leading coefficient* of a polynomial is the coefficient of the term of the polynomial that determines the degree.

**Exercise:** Determine the leading coefficients of the above expressions that are polynomials.

End Behavior of Polynomial Functions

The end behavior of a polynomial function is completely determined according to the following chart.

<table>
<thead>
<tr>
<th>degree</th>
<th>leading coefficient</th>
<th>end behavior towards the left</th>
<th>end behavior towards the right</th>
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<tbody>
<tr>
<td>even</td>
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<tr>
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<td>negative</td>
<td>positive</td>
<td>negative</td>
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</tbody>
</table>

**Exercise:** Determine the end behavior of the above expressions that are polynomials.