Oscillating Integrals

Stokes phenomenon describes discontinuous behavior of asymptotic expansions of “oscillating integrals”, i.e. integrals given by

$$I(a; k) = \int_C \exp(ikP(a, z)) \omega$$

(here $P$ is a meromorphic function of complex variables $z$ depending on parameters $a; k$ - large real parameter, $C$ an appropriate contour of integration). An archetypal example of such oscillating integrals is (a version of) the Airy function,

$$Ai(a; k) = \int \exp(ik(z^3 + az))dz$$

Stokes phenomena, despite their analytic guise have purely topological roots. The problem of understanding the combinatorial and topological structure of Stokes sets for the case when $P(a, z) = \sum_k a_kz^k$ is a polynomial was initiated by Sir Michael Berry in [1], and turned out (see [2]) to be closely connected to a family of convex polyhedra, interpolating between famous Stasheff’s associahedra and lowly cubes.

The problem at hand this summer will be to work out the combinatorics of slightly more complex classes of functions, say those given by

$$a_{-1}/z + P(a, z),$$

the simplest Laurent polynomials...

References

[1] M. V. Berry, Stokes phenomenon; smoothing a victorian discontinuity, Publications Mathematiques de L’IHS Volume 68, Number 1, 211-221.