Three-periodic orbits on hyperbolic plane

Consider what is called the classical (Birkhoff) billiard problem: a particle moves in the planar convex domain $D$ along the straight lines interacting with the boundary $\partial D$ according to the Fermat’s law “angle of reflection is equal to the angle of incidence”. In mathematical physics it is important to understand how many periodic orbits can be present in a planar billiard. *Periodic orbits are those which repeat themselves after finitely many bounces and the number of bounces before the repetition is the period.*

It has been generally believed that periodic orbits constitute a set of zero measure, but the answer is known only for periods 2 and 3, see [1] or for a much simpler proof [2]. The sub-Riemannian geometry methods have been used for billiard problems in [3] and for outer billiard problem [4].

It is natural to consider this problem in other geometries. In spherical geometry, billiards with positive measure of 3-period orbits have been classified in [5]. It turns out that the boundary obtained by intersecting the sphere with one of the octants will have only 3-period orbits. All other spherical billiards with positive measure of 3-period orbits are obtained from this canonical example.

For this summer we plan to study the analogous problem for 3-period orbits on hyperbolic plane. Our hope is to use sub-Riemannian geometry methods to prove that on the hyperbolic plane all billiards have zero measure of 3-period orbits.

1 Wojtkovski method

It is natural first to try a simpler approach developed by Wojtkovsky in [2] where he proved that 3-period orbits in classical billiard constitute set of zero measure. He used Jacobi fields for his argument and this should be possible to extend to other geometries (in particular to hyperbolic geometry). You only need to read sections 1,2 and 4 in [2] which are relevant for us. The first task is to derive the relations like formulas (9)-(11) for the hyperbolic plane.

References


