2.1 (8) (a) Let \( v = ru \)

\[ u_r = \frac{v}{r} + ru_r \]

\[ u_{rr} = 2u_r + ru_{rr} \]

\[ u_{tt} = c^2 \left( u_{rr} + \frac{2}{r} u_r \right) \]

\[ ru_{tt} = c^2 \left( ru_{rr} + 2u_r \right) \]

\[ \nabla \left( ru_{tt} \right) = c^2 u_{rr} \]

(b) The general solution

\[ v = \varphi (r-c t) + \psi (r+c t) \]

\[ u = \frac{1}{r} \varphi (r-c t) + \frac{1}{r} \psi (r+c t) \]

(c) Initial conditions:

\[ u(r, 0) = \varphi (r), \quad u_t (r, 0) = \varphi (r) \]

Then \( v(r, 0) = r \varphi (r), \quad u_t (r, 0) = r \psi (r) \)
Using (8), we have

\[ u(r, t) = \frac{1}{2} \left[ (r+c_t)\varphi(r+c_t) + (r-c_t)\varphi(r-c_t) \right] \]

\[ + \frac{1}{2c} \int_{r-c_t}^{r+c_t} s \varphi(s) \, ds \]

and then since \( \varphi = r \varphi \)

\[ w(r, t) = \frac{1}{2r} \left[ (r+c_t)\varphi(r+c_t) + (r-c_t)\varphi(r-c_t) \right] \]

\[ + \frac{1}{2cr} \int_{r-c_t}^{r+c_t} s \varphi(s) \, ds \]

2.2 (1): For \( \varphi \equiv 0, \varphi = 0, \)

Energy = 0, then
\[ \int_{-\infty}^{\infty} u_{x}^{2} \, dx + T \int_{-\infty}^{\infty} u_{x}^{2} \, dx = 0 \]

but, \( T, p > 0 \), then \( \int_{-\infty}^{\infty} u_{x}^{2} \, dx = 0 \)

By vanishing theorem,

\[ u(x, t) = f(t). \]

Similarly, \( \int_{-\infty}^{\infty} u_{x}^{2} \, dx = 0 \)

and then, \( u(x, t) = f(x) \).

Thus, \( u(x, t) = \text{Constant} \),

which must be "0" by i.e.
4. \[ u(x,t) = f(x-t) + g(x+t) \]

Both \( f(x-t) \) and \( g(x+t) \) satisfy the identity.

Verify for \( f(x-t) \):

\[
\underbrace{f(x+h-t-k) + f(x-h-t+k)} = \underbrace{f(x+k-t-h) + f(x-k-t+h)}
\]

The identity indeed holds as the underlined terms match.

Verification for \( g(x+t) \) is similar.
(2.3) \( B \):

(a) \( u(x, t) > 0 \) in the interior by the strong minimum principle.

(b) \( \mu(t) = u(x(t), t) \)

\[ \frac{d\mu}{dt} = u_x x(t) + u_t \quad u_x = 0 \]

Since \( u(x(t), t) \) is maximum in \( x \) and \( u_t = u_{xx} \leq 0 \) for the
same reason \( u(x(\tau), \tau) \) is max \( u(x, \tau) \).

Thus, \( \frac{dM}{dt} \leq 0 \).