Problem # 1. Let \( y : \mathbb{R}^1 \rightarrow \mathbb{R}^n \) and let \( y \) be differentiable. Prove the inequality

\[
\frac{d}{dt} |y(t)| \leq \left| \frac{d}{dt} y(t) \right|
\]

(Hint: Use the definition of the derivative and the triangle inequality \( |a + b| \leq |a| + |b| \).)

Problem # 2. Consider IVP \( \dot{x} = f(x), x(0) = 0 \) with

\[
f(x) = x \ln \frac{1}{|x|}, \text{ if } x \neq 0
\]

and \( f(0) = 0 \).

(a) Prove that IVP possesses a unique solution \( x(t) \equiv 0 \)

(b) Verify that \( f(x) \) is not a Lipschitz function.

This example shows that the equation may possess uniqueness property even if the Lipschitz condition is not satisfied.

Problem # 3. Consider the equation

\[
\dot{x} = \frac{1}{\cosh(x)}
\]

on the real line.

(a) Find the solution of the initial value problem \( x = \phi(t, x^0) \).

(b) Verify that \( \phi \) is a group of transformations on \( \mathbb{R}^1 \).

(c) Check if \( \phi \) is a group of diffeomorphisms.

Problem # 4 Consider the second order equation \( \ddot{x} - k^2 x = 0 \) with \( k \neq 0 \) as a system of first order equations. Find the solution of the initial value problem. Verify the flow properties of the solution directly, i.e. show the solutions form a one-parameter group of transformations on \( \mathbb{R}^2 \).