1. Use the definitions of sinh $x$ and cosh $x$ to show that
   
   (a) $\cosh^2 x - \sinh^2 x = 1$
   
   (b) $\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$ [Hint: Start with the right side.]

2. Suppose $\sinh x = \frac{4}{3}$. Use the identity $\cosh^2 x - \sinh^2 x = 1$ and the definitions of the hyperbolic functions to find the other five hyperbolic functions of $x$.

3. Show that $f(x) = \sinh x$ is an odd function and $g(x) = \cosh x$ is an even function. [Hint: What are the definitions of odd and even?]

4. Evaluate $\lim_{x \to \infty} \frac{\sinh x}{e^x}$.

5. Use the definitions of the hyperbolic functions to show that
   
   (a) $\frac{d}{dx} \sinh x = \cosh x$
   
   (b) $\frac{d}{dx} \cosh x = \sinh x$

6. A telephone line hangs between two poles 14 m apart in the shape of the catenary $y = 20 \cosh(x/20) - 15$, where $x$ and $y$ are measured in meters. Note that position $x = 0$ is halfway between the poles.
   
   (a) Find the slope of this curve where it meets the right pole.
   
   (b) Find the angle $\theta$ between the line and the pole.

7. Show that $\frac{d}{dx} \tanh x = \text{sech}^2 x$.

8. Show that $\frac{d}{dx} \arctan(\tanh x) = \text{sech} (2x)$. [Hint: Use the results from Problems 7 and 1b.]

9. Evaluate the following integrals.
   
   (a) $\int \sinh \frac{x}{5} \, dx$
   
   (b) $\int \text{sech}^2 (2t) \, dt$
   
   (c) $\int 6 \cosh \left(\frac{x}{2} - \ln 3\right) \, dx$
   
   (d) $\int_1^4 \frac{8 \sinh \sqrt{y}}{\sqrt{y}} \, dy$
\[ (e) \int_{-\pi/4}^{\pi/4} \cosh(\tan \theta) \sec^2 \theta \, d\theta \]

10. Find derivatives of the following functions.

(a) \( f(t) = t^2 \tanh \frac{1}{t} \)
(b) \( g(x) = 6 \sinh \frac{x}{3} \)
(c) \( h(x) = \ln(\cosh x) - \frac{1}{2} \tanh^2 x \)
(d) \( f(y) = \sinh(\ln y) \)

11. Show that \( \frac{d}{dx} \sqrt{\frac{1 + \tanh x}{1 - \tanh x}} = \frac{1}{2} e^{x/2} \).

12. The Gateway Arch in St. Louis was designed by Eero Saarinen and was constructed using the equation \( y = 211.49 - 20.96 \cosh 0.03291765x \) for the central curve of the arch, where \( x \) and \( y \) are measured in meters and \( |x| \leq 91.20 \).

(a) What is the height of the arch at its center?
(b) At what points is the height 100 m?
(c) What is the slope of the arch at the points where the height is 100 m?