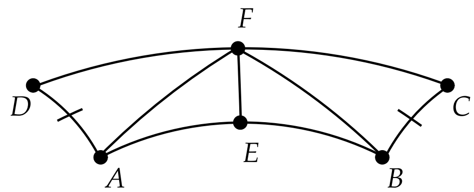
MATH 402 Worksheet 8

Friday 10/21/16

In the project about Saccheri quadrilaterals, you learned that in a Saccheri quadrilateral as in the figure below (with the angles at A and B being right), you also have:

- The angles at C and D are acute and equal, and
- The lines AB and CD are ultraparallel to each other, as they share a common perpendicular, which is the perpendicular bisector to both segments $\overline{AB}$ and $\overline{CD}$.



There is another special type of a quadrilateral in hyperbolic geometry: that's a quadrilateral with 3 right angles, which is called a Lambert quadrilateral.

(1) Show that in a Lambert quadrilateral, the fourth angle must be acute. As a consequence, we get that rectangles do not exist in hyperbolic geometry.
Hint: First prove that you can "double" a Lambert quadrilateral to get a Saccheri quadrilateral.

(2) Show that the angle sum of a hyperbolic triangle $\triangle ABC$ is strictly less that $180°$, using the following strategy:
- Assume $\triangle ABC$ is a right triangle, with right angle at A. Take the midpoint M of $\overline{BC}$, and drop a perpendicular to AC from M at D. Find a Lambert quadrilateral with vertices A, B, D.
- Use knowledge about the angles of this Lambert quadrilateral to show the angle sum of $\triangle ABC$ is less than $180°$.
- Given a general triangle (not necessarily right), split it into two right triangles and use the above.

(3) Show that the angle sum of any quadrilateral is less than $360°$.

Given $\triangle ABC$, the difference between $180°$ and the angle sum of $\triangle ABC$ is called the *defect* of $\triangle ABC$. Given a quadrilateral $ABCD$, its defect is defined to be the difference between $360°$ and its angle sum.

(4) Use defects to show that if two triangles have congruent corresponding angles, then they are congruent.

[**Bonus:**] Use Saccheri quadrilaterals to show that parallel lines cannot be everywhere equidistant.